Uncertainty, misaligned expectations, and bond term premium measures*

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ABSTRACT: This paper shows that inflation expectations and those embedded in short-term interest rate expectations as reported in the Survey of Professional Forecasters show evidence of misaligned expectations. This misalignment seems to have been substantial in recent times, featuring a low correlation between inflation and the policy rate (i.e. the Gibson paradox). This empirical evidence motivates an alternative explanation, based on uncertainty rather than risk, of the bond term premium measures found in the literature. This paper estimates an *expectational* term premium driven by misaligned short-term interest rate expectations from a medium-scale DSGE model, which introduces uncertainty by assuming adaptive learning (AL) with *discretional beliefs*. The estimated 10-year expectational term premium shares important features with the corresponding term premium measures obtained in the literature using no-arbitrage affine term structure models. Thus, the expectational term premium is sizable, highly persistent, and mildly countercyclical. More important, this expectational AL term premium is highly correlated with term premium measures obtained from no-arbitrage affine models in the most recent period studied. In short, the estimation results suggest that model uncertainty provides an important channel for explaining the bond premium measures lately by introducing a potential misalignment of short-term interest expectations with inflation expectations.

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1 Introduction

The bond term premium is defined as the wedge between a long-term yield and the average of policy rate expectations—i.e. the yield implied by the expectation hypothesis of the term structure (EHTS)— over the life of the long-term bond. Thus, the size of the term premium signals the effectiveness of the policy rate (and the forward guidance in shaping policy rate expectations) in affecting the yield curve. The term premium is typically viewed as the additional return (a risk premium) that investors demand to compensate them for the risk associated with a long-term bond. The term premium is not observable, but it is typically estimated using no-arbitrage affine models (Kim and Wright, 2005; and Adrian, Crump, and Moench, 2013) featuring a small number of risk (pricing) factors, in which the parameters linking linearly these factors and bond yields are restricted to eliminate any source of arbitrage opportunities. The rational expectations DSGE literature has struggled to generate term premiums with the size and the appropriate dynamic features of those estimated using no-arbitrage affine models (Rudebusch and Swanson, 2012; Lucas, 2003).

Rather than seeing the term premium as only a risk premium, I argue that the estimated term premium measures obtained from no-arbitrage affine models may incorporate uncertainty, in addition to risk, in their latent estimated factors.[1] In order to analyze this hypothesis, this paper characterizes an endogenous *expectational* adaptive learning term premium, defined as the wedge between the (optimal) nominal yield associated with the j-period maturity bond determined by a DSGE model and the yield implied by the EHTS, where the two yields are obtained under the assumption of adaptive learning (AL) with *discretional beliefs* described below. In contrast to the standard view of the bond term premium as a compensation for risk, this expectational term premium is driven by model uncertainty mod-

[1] A standard distinction between these two concepts can be described as follows. *Risk* refers to a scenario under which the decision outcomes and their probabilities of occurrences are known to the decision-maker, while *uncertainty* refers to a scenario under which that information is not available to the decision-maker (Knight, 1921). Under rational expectations, decision-makers are usually assumed to know the true data generating process, so uncertainty is ruled out. However, uncertainty may come into play when departures from the rational expectations assumption are considered.

eled here through the AL assumption, which results in a misalignment between the inflation expectations of agents and those embedded in short-term interest (policy rate) expectations.[2] Interestingly, I find that average survey-based measures of inflation expectations and short-term interest rate expectations as reported in the Survey of Professional Forecasters (SPF) show evidence of misaligned interest rate expectations in recent times. More precisely, there is a misalignment between survey-based 1-year-ahead inflation forecasts and those inflation forecasts implied by survey-based short term policy rate forecasts computed through the lens of a log-linearized asset pricing model. This is a novel finding, which challenges the effectiveness of forward guidance (i.e. timely communication from central banks about the future course of monetary policy) in leading the public's inflation expectations.

Seeking to provide a novel approach to explaining the bond term premium based on bounded rationality and model uncertainty, this paper estimates a term premium from an estimated DSGE model extended with the term structure of interest rates, assuming adaptive learning (AL) expectations. In this AL framework, agents do not know the structure of the economy, so they face a major source of uncertainty which is ignored in standard approaches built on the rational expectations (RE) hypothesis. Following Aguilar and Vázquez (2021) and Vázquez and Aguilar (2021), I assume that AL expectations are based on small forecasting models as in Slobodyan and Wouters (2012a,b), where the expectation of each forward-looking variable in the DSGE model is described as a least-squares projection on a small information set given by the contemporaneous and first-lag values of the forward-looking variable.[3] Moreover, the small forecasting models are augmented with term structure information, and it is assumed that agents' expectations of a forward-looking variable at short-term forecast horizons do not necessarily lead to *consistent* predictions of the variable far into the future as implied by the law of iterated expectations used in other approaches to

[2] The approach followed here is in sharp contrast to a related paper by Vázquez and Aguilar (2021), who characterize an exogenous AL term premium as the wedge between the actual yield of the j-period maturity bond and the yield implied by the EHTS, which means that they then ignore misaligned expectations.

[3] This approach is associated with the broad class of restricted perception equilibriums, where agents consider a misspecified model but form their beliefs optimally given the misspecification (Sargent, 1991; Hommes and Sorger, 1998; Evans and Honkapohja, 2012).

AL.[4] I refer to these AL expectations, characterizing a precise form of uncertainty, as *discretional beliefs*. These assumptions on beliefs are in line with the evidence provided by Froot and Ito (1989) that short term investors' expectations are not consistent with their long-term expectations. In particular, they find that short term investors' expectations overreact to current shocks when compared with their long-term expectations. They are also in line with how panelists in the SPF behave in changing their methods with the forecast horizon as pointed out in Stark (2013).[5] Most importantly, abandoning the law of iterated expectations is crucial in characterizing a potential misalignment of short-term interest rate expectations, which generate the *expectational* AL term premium suggested in this paper.

Readers might wonder whether the small forecasting models used in the AL approach considered here are too flexible by adding degrees of freedom on the information set that agents are using in their forecasting models. However, this concern does not seem to be an important empirical issue for several reasons (some of them already pointed out in Slobodyan and Wouters, 2012): First, the data is shown to be informative on the small forecasting models considered in the AL approach followed here. Thus, the additional flexibility provided by simple forecasting models is optimally used to fit the overall model on the data while avoiding ending up in an unidentified model. Moreover, larger forecasting models include more variables in the information set, and thus feature higher parameter uncertainty and potentially more instability in the updating process. This overfitting problem certainly introduces a bias toward smaller forecasting models. It turns out that a small forecasting model based only on term spread information mainly captures learning dynamics, while the type

[4] More precisely, I consider an AL approach based on direct multi-step forecasting (Bhansali, 2002; Jordà, 2005) to characterize the multi-period-ahead expectations of forward-looking variables that show up when the DSGE model is extended with the term structure of interest rates. This contrasts with the approach followed in the related AL literature, which uses iterated forecasts built in general on a misspecified model. As discussed by Jordà (2005), among others, direct forecasts associated with multi-period horizons outperform iterated forecasts in dealing with misspecified models since misspecification errors are compounded with the forecast horizon. Direct multi-step forecasting introduces additional flexibility which is not taken into account when iterated forecasts are considered, and that extra flexibility results in a better model fit (Aguilar and Vázquez, 2021).

[5] The failure of the law of iterated expectations could also show up under RE whenever investors have heterogeneous information sets. An example of this approach is followed in Barillas and Nimark (2019).

of small forecasting model based on second order autoregressive processes used in Slobodyan and Wouters (2012) plays a rather minor role in our estimated DSGE model under AL with arbitrary beliefs. As pointed out in Aguilar and Vázquez (2021), simple forecasting models based only on term structure information perform well because the time-varying intercepts in them capture the long-run features of data well, such as the downtrend exhibited by inflation and the short-term interest rate in the early 1980s. Meanwhile, the time-varying learning coefficients associated with term spreads capture short-run macroeconomic fluctuations relatively well (for instance, those featuring the growth rates in consumption and investment). Second, the estimates of structural parameters and the identification of shocks do not seem to be very sensitive to the forecasting model considered. Third, average survey-based measures of short-term interest expectations and inflation expectations obtained from the SPF are also included in the set of observables used in the estimation procedure in order to discipline the learning parameters featuring small forecasting models and their updating processes. Hence, the discretional beliefs identified in the estimation procedure are to some extent aligned with survey-based expectations. Finally, it is important to emphasize that the term premium measures obtained from estimated no-arbitrage affine models do not enter into the estimation procedure used here; rather, they are only used to assess the expectational AL term premium generated as an estimated latent variable in the estimation procedure. Hence, both the estimated small forecasting models and the potential failure of the law of iterated expectations are assumptions which are empirically falsifiable by assessing the ability of the estimated AL term premium measure to reproduce the dynamic features characterizing the term premiums obtained from no-arbitrage affine models.

The AL-DSGE model is estimated using US quarterly data for 1983:2-2017:3. The estimation results show that the estimated expectational term premium obtained under AL with discretional beliefs shares some features with those estimated in the related literature, such as those suggested by Kim and Wright (2005) and Adrian, Crump, and Moench (2013) using no-arbitrage affine models. Thus, the estimated expectational AL term premium is sizable, highly persistent, and mildly countercyclical; but it shows a rather low correlation with term premium measures obtained from no-arbitrage affine models. However, the picture changes quite dramatically if attention is focused on recent times. Thus, the estimated 10-year expectational AL term premium is highly correlated (0.82) with the Adrian-Crump-Moench term premium and more moderately correlated with the Kim-Wright term premium (0.71) for 2001:1-2017:3, while the correlation between the term premiums from the two affine models is 0.80 in that period. Hence, the misaligned policy rate expectations captured by the expectational AL term premium with discretional beliefs seem to be substantial in recent times, and the Adrian-Crump-Moench model is somewhat better able to capture this decoupling of nominal expectations than the Kim-Wright model. Moreover, the evidence found on misaligned policy rate expectations challenges the effectiveness of recent forward guidance as a monetary policy tool to some degree. This misalignment seems to be linked to the low correlation between inflation and the short-term nominal interest rate (i.e. the Gibson paradox) found in Cogley, Sargent, and Surico (2012) and Casares and Vázquez (2018), which shows the limitations at controlling inflation through an orthodox monetary policy based on policy rate adjustments.

Variance decomposition analysis shows that wage markup shocks, investment-specific technology shocks, and monetary policy shocks between them explain around 95% of the fluctuations in the 10-year expectational AL term premium, and hence the long-term expectation misalignment, but their relative contributions change quite dramatically depending on the forecast horizon considered in the variance decomposition. Thus, the main drivers of short-term fluctuations in the expectational AL term premium are monetary policy shocks (67.3%), while the contributions of wage markup shocks and investment-specific technology shocks are much smaller at 22.7% and 3.7%, respectively. These results change dramatically when the unconditional variance decomposition is analyzed: Investment technology shocks then explain 85.1% of the long-term fluctuations in the expectational AL term premium, while the contributions of policy shocks and wage markup shocks drop substantially to 6.8%

and 4.5%, respectively.

In sum, these findings suggest that model uncertainty, illustrated in this paper by the presence of a representative agent who forms expectations using an AL scheme with discretional beliefs, provides an alternative interpretation of the bond premium measures found in the literature which is in contrast to other features —such as consumer preferences featuring high-level risk aversion— assumed in DSGE frameworks to generate sizable term premium fluctuations. In particular, these findings provide empirical support for the hypothesis put forward in the literature (Barillas, Hansen, and Sargent, 2009; Rudebusch and Swanson, 2012; among others) that model uncertainty is an alternative to the unpleasantly large risk aversion parameters of 50 or 100 needed in RE-DSGE models to fit the data as criticized in Lucas (2003).

The rest of the paper is structured as follows. Section 2 reviews some issues in the term premium and AL branches of literature. Section 3 describes a simple model to illustrate how misaligned nominal expectations result in an expectational term premium and provides some preliminary evidence based on survey-based forecasts reported in the SPF. Section 4 considers a DSGE model extended with the term structure of interest rates and derives an expectational term premium under AL with discretional beliefs. Section 5 shows the estimation results of the DSGE model and discusses their implications. Section 6 analyzes the dynamic features of the AL term premium estimated here in comparison with those estimated in the related literature. Section 7 concludes.

2 Related literature

This section links the contribution of this paper to two branches of literature.

Financial premium literature

Structural macroeconomic literature has broadly taken two approaches to explaining financial premiums, as summarized in Kliem and Meyer-Gohde (2021). One approach incorporates financial market frictions and/or segmented markets (e.g. Gertler and Karadi, 2011; Carlstrom, Fuerst, and Paustian, 2017; Sims and Wu, 2021) that restrain financial market participants and generate premiums as spreads between market rates (e.g. of different maturities) and the risk-free, short-term rate. Such premiums are risk-neutral expected losses from, say, default, which leads to the second approach taken in the literature: In that second approach, time-varying term premiums result from the pricing via the stochastic discount factor of endogenous conditional heteroskedasticities generated by the nonlinearities in the model (e.g. Rudebusch and Swanson, 2012). This approach views term premiums on medium- and long-term nominal bonds as compensations for the inflation and consumption risks faced by investors over the lifespans of bonds. There is a large body of literature that finds that estimated 10-year term premium measures obtained from no-arbitrage affine term structure models are sizable, persistent, and fluctuate significantly over time (e.g. Gürkaynak and Wright, 2012 and references therein). However, standard macroeconomic DSGE models based on the RE hypothesis find it hard to explain term premium dynamics. For instance, Rudebusch and Swanson (2012) make progress by combining several features: (i) Epstein-Zin preferences (Epstein and Zin, 1989), so risk aversion can be modeled independently from the intertemporal elasticity of substitution; (ii) a third-order approximation;[6] (iii) both long-run real (as in Bansal and Yaron, 2004) and nominal risks; and (iv) a huge risk aversion. Using similar RE-DSGE models, Dew-Becker (2014), Kliem and Meyer-Gohde (2021), and Amisano and Tristani (2023) estimate term premium measures with rather distinctive empirical features.[7] In spite of the progress made in this literature, all these (estimated/calibrated) DSGE models rely on an unpleasantly large risk aversion parameter, which is needed to overcome the lack of model uncertainty implied by the RE hypothesis in the characterization of

[6]In this class of model, a first-order approximation eliminates the term premium entirely due to the well-known property of certainty equivalence in linearized RE models. Indeed, a third-order approximation is needed to obtain a (model-based) time-varying term premium (Rudebusch and Swanson, 2008).

[7]Thus, the latter two reproduce the mild downward trend displayed by the estimated term premium measures obtained from no-arbitrage affine term structure models since the early 1980s (for instance, Adrian, Crump, and Moench, 2013) quite well. However, the term premium estimated by Dew-Becker (2014) shows an upward trend, at least at the start of the Great Moderation period, which is in contrast to other term premium measures.

term premium dynamics. One important exception is the paper by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018): They use a GMM method to estimate the pruned state-space system for a third-order perturbation approximation to a New Keynesian model with Epstein–Zin preferences that includes feedback effects from long-term bonds to the real economy, enabling them to match the level and variability of the 10-year term premium with a low relative risk aversion of 5.

Adaptive learning literature

As pointed out by Adam and Marcet (2011), AL literature takes the first-order optimality conditions that emerge under RE and replaces the conditional RE operator, E_t, in those optimality conditions by an AL conditional expectations operator, E_t^{AL}. Standard AL approaches typically assume that the law of iterated expectations implied by the RE hypothesis also holds for the subjective expectations of the representative agent under AL (Honkapojha, Mitra, and Evans, 2003). Imposing the law of iterated expectations on AL conditional expectations has led to some conflicting results, as described in Adam and Marcet (2011). For instance, Adam, Marcet, and Nicolini (2016) consider the one-step-ahead asset pricing equation $P_t = \delta E_t^{AL}(P_{t+1}+D_{t+1})$ and show that a variety of stylized asset pricing facts can be explained if agents are learning about future price behavior. By contrast, Timmermann (1996) and others impose the law of iterated expectations and study the implied relationship between the stock price and the expected discounted sum of dividends ($P_t = E_t^{AL} \sum_{j=1}^{\infty} \delta^j D_{t+j}$), which results in a much weaker impact on stock prices from dividend learning behavior. Evans and Honkapohja (2003) are much closer to the monetary DSGE framework considered in this paper: They consider AL in one-step-ahead Euler optimality conditions. Preston (2005) shows that learning outcomes also differ in models of this type when the one-step-ahead Euler optimality conditions are iterated forward based on the law of iterated expectations. Interestingly, Adam and Marcet (2011) strongly argue that (one-step-ahead) Euler equations under AL—rather than optimality conditions derived through the application of the law of iterated expectations— are less ad-hoc from a microfoundation perspective of the consumption-based asset pricing model. More precisely, Adam and Marcet (2011) suggest that the law of iterated expectations may fail under realistic circumstances such as imperfect market knowledge and the lack of common knowledge on agents' beliefs and preferences. Thus, a marginal agent that prices a stock or bond in actual markets is changing with time, and given that agents hold heterogeneous beliefs, the equilibrium price is determined by expectations based on different probability measures in each period. As a result, the iteration forward on the one-step-ahead pricing equation may not properly represent agents' optimal pricing decisions. Adam, Marcet, and Beutel (2017) make further progress on these asset price determination issues.[8]

This paper is also related to recent papers by Aguilar and Vázquez (2021) and Vázquez and Aguilar (2021) that estimate log-linear DSGE models under AL using the type of small forecasting models considered here. In sharp contrast to those two papers, here the EHTS is not imposed in determining the long-term bond yields. This results in an *endogenous* expectational bond term premium defined as the difference between the yield implied by the standard consumption-based asset pricing equation under AL and the yield implied by the EHTS.[9]

[8]Kozicki and Tinsley (2005) is an early paper that considers adaptive expectations (rather than the AL used here) to characterize term structure dynamics. More specifically, its authors consider a model where the EHTS is imposed (i.e. long-term interest rates are given by agents' beliefs about expected average future short-term rates), but in which adaptive beliefs are determined by agents' perceptions of the central bank's long-run inflation target. As discussed in Gürkaynak and Wright (2012), there is a related literature that seeks to explain term structure anomalies in terms of shifting perceptions of the central bank's long-run inflation target. More recently, Sinha (2015, 2016) also investigates the implications of AL for the yield curve. However, her approach to AL is rather different from the one followed here, as discussed below. Moreover, her papers do not address the implications of AL for the estimated bond term premium.

[9]This paper also considers an extended sample period (1983-2017) that includes the Great Recession, whereas the other two papers consider a different sample period that includes the Stagflation period of the 1970s and the early 1980s and finishes at the end of the Great Moderation period (around 2008).

3 A simple model of misaligned expectations and some preliminary evidence

The standard consumption-based asset pricing equation associated with each maturity is obtained from the first-order conditions that characterize the optimal decisions of the representative consumer endowed with discretional beliefs:

$$E_t^D\left[M_t^{\{j\}}\frac{\left(1+R_t^{\{j\}}\right)^j}{\prod_{k=1}^{j}\left(1+\pi_{t+k}\right)}\right]=1,\ for\ j=\ 1,\ 2,\ ...,\ n,$$

where E_t^D defines a discretional expectations operator,[10] $M_t^{\{j\}}=\beta^j\frac{U_{C,t+j}}{U_{C,t}}$ is the stochastic discount factor (i.e. the pricing kernel) associated with a j-period maturity bond, β is the subjective discount factor, $U_{C,t}$ denotes the marginal utility of consumption at time t, and C_t, π_t, and $R_t^{\{j\}}$ denote consumption, the rate of inflation, and the nominal yield associated with the j-period maturity bond, respectively. The set of non-linear optimality conditions clearly shows that current consumption, C_t, and the yields associated with alternative maturity bonds, $R_t^{\{j\}}$, which are priced at time t, are determined by the expected paths of future consumption, and inflation. $R_t^{\{j\}}$ is known at time t, so the set of the optimality conditions can be written as

$$\left(1+R_t^{\{j\}}\right)^j E_t^D\left[M_t^{\{j\}}\frac{1}{\prod_{k=1}^{j}\left(1+\pi_{t+k}\right)}\right]=1,\ for\ j=\ 1,\ 2,\ ...,\ n, \tag{1}$$

Using the expression for the conditional covariance between the pricing kernel, $M_t^{\{j\}}$, and the inverse of inflation over the lifetime of the bond, $\frac{1}{\prod_{k=1}^{j}(1+\pi_{t+k})}$, equation (1) can be written as

$$E_t^D\left(M_t^{\{j\}}\right)E_t^D\left[\frac{1}{\prod_{k=1}^{j}\left(1+\pi_{t+k}\right)}\right]+cov_t\left(M_t^{\{j\}},\frac{1}{\prod_{k=1}^{j}\left(1+\pi_{t+k}\right)}\right)=\left(1+R_t^{\{j\}}\right)^{-j}. \tag{2}$$

[10] By a "discretional expectations operator", I mean an expectations operator that deviates from the standard RE expectations operator and, in addition, *does not* satisfy the law of iterated expectations.

In contrast to the related literature, and in order to focus on the alternative interpretation of the term premium based on expectation misalignment driven by model uncertainty, I ignore here the covariance term which captures the standard interpretation of a term premium as a compensation for risk—i.e. I impose that $cov_t\left(M_t^{\{j\}}, \frac{1}{\prod_{k=1}^{j}(1+\pi_{t+k})}\right) = 0$. This restriction can simply be imposed by taking (natural) logs in equation (1) and ignoring Jensen's inequality of expectation as follows:

$$jR_t^{D\{j\}} + E_t^D m_t^{\{j\}} - \sum_{k=1}^{j} E_t^D \pi_{t+k} = 0,$$

or

$$R_t^{D\{j\}} = \frac{1}{j}\left(\sum_{k=1}^{j} E_t^D \pi_{t+k} - E_t^D m_t^{\{j\}}\right), \tag{3}$$

where $m_t^{\{j\}} = log\left(M_t^{\{j\}}\right)$. Notice that I have added the superscript "D" to the nominal yield of the j-period maturity bond to emphasize that this yield is obtained under discretional learning.

I define the *expectational* term premium of the j-period bond as

$$TP_t^{D\{j\}} \equiv R_t^{D\{j\}} - \frac{1}{j}\sum_{k=0}^{j-1} E_t^D R_{t+k}^{\{1\}}. \tag{4}$$

Substituting equation (3) into (4) gives

$$TP_t^{D\{j\}} = \frac{1}{j}\left(\sum_{k=1}^{j} E_t^D \pi_{t+k}\right) - \frac{1}{j}\left(\sum_{k=0}^{j-1} E_t^D R_{t+k}^{\{1\}} + E_t^D m_t^{\{j\}}\right). \tag{5}$$

According to equation (3), the expectation of the (log-) pricing kernel, $\frac{1}{j}E_t^D m_t^{\{j\}}$, is the negative of the ex-ante real return of the j-period bond, so the expectational term premium, $TP_t^{D\{j\}}$, can then be intrepreted as a misalignment between the average of expected inflation, $\frac{1}{j}\left(\sum_{k=1}^{j} E_t^D \pi_{t+k}\right)$, and the average of (expected) inflation embedded in the short-term policy rate expectations,

$$\frac{1}{j}\left(\sum_{k=0}^{j-1} E_t^D R_{t+k}^{\{1\}} + E_t^D m_t^{\{j\}}\right),$$

over the lifetime of the bond (i.e. the difference between the average of policy rate expectations and the expected real return, $-\frac{1}{j}E_t^D m_t^{\{j\}}$).

Assuming a standard additive-separable power utility function in consumption and leisure, the marginal utility of consumption is

$$U_{C,t} = (C_t)^{-\sigma},$$

where σ denotes the constant relative risk aversion parameter. This implies that the log of the pricing kernel is given by

$$m_t^{\{j\}} = log\left(M_t^{\{j\}}\right) = log\left(\beta^j \frac{U_{C,t+j}}{U_{C,t}}\right) = jlog\left(\beta\right) - \sigma\left(logC_{t+j} - logC_t\right).$$

Substituting this expression in (5) gives

$$TP_t^{D,\{j\}} = \frac{1}{j}\left(\sum_{k=1}^{j} E_t^D \pi_{t+k}\right) - \left[\frac{1}{j}\left(\sum_{k=0}^{j-1} E_t^D R_{t+k}^{\{1\}}\right) - \frac{\sigma E_t^D\left(logC_{t+j} - logC_t\right)}{j} + log\left(\beta\right)\right], \quad (6)$$

where $\frac{E_t^A(logC_{t+j} - logC_t)}{j}$ captures the expected average of the growth rate of consumption over the life of the bond.

Figure 1 shows evidence on the decoupling of average survey-based measures of inflation expectations reported in the SPF and those inflation expectations embedded in the average survey-based measures of the short-term interest rate expectations also reported in the SPF. More precisely, the upper graph in this figure summarizes this decoupling by showing the SPF 1-year expectational term premium, $TP_t^{SPF,\{4\}}$, defined as $4 \times \left\{\frac{1}{4}\left(\sum_{k=1}^{4}\pi_{t+k}^{SPF}\right) - \left[\frac{1}{4}\left(\sum_{k=0}^{3} R_{t+k}^{\{1\},SPF}\right) - \frac{\sigma[logC_{t+4} - logC_t]^{SPF}}{4} + 100 \times log\left(\beta\right)\right]\right\}$ (black line), [11]

[11] To keep the same units across the four components of this expectational term premium, I first compute inflation and the short-term nominal rate in quarterized units, given that the SPF reports them in annualized

where the SPF inflation expectations accumulated over 1 year are given by $\sum_{k=1}^{4} \pi_{t+k}^{SPF}$ (blue line), and $\pi_{t,t+k}^{SPF}$ denotes the k-period-ahead expectations for the inflation rate reported in the SPF; $R_{t,t+k}^{\{1\},SPF}$ denotes the k-period-ahead expectations for the one-quarter nominal interest rate reported in the SPF, so $\sum_{k=0}^{3} R_{t+k}^{\{1\},SPF}$ (grey line) denotes the SPF short-term nominal rate expectations accumulated over 1 year; $\frac{[logC_{t+4} - logC_t]^{SPF}}{4}$ is the expected average consumption growth rate over the 1-year forecast horizon reported in the SPF; $\sigma = 1.5$ and $\beta = 0.9982$.[12] Thus, $\left[\left(\sum_{k=0}^{3} R_{t+k}^{\{1\},SPF}\right) - \sigma \left[logC_{t+4} - logC_t\right]^{SPF} + 400 \times log\left(\beta\right)\right]$ denotes the inflation expectations path embedded in the short-term policy rate expectations over the 1-year forecast horizon (red line), and $\sigma \left[logC_{t+4} - logC_t\right]^{SPF} - 400 \times log\left(\beta\right)$ denotes the corresponding 1-year ex-ante real interest rate (green line). Notice that while SPF inflation expectations (blue line) remain stable in the second half of the sample period, the inflation expectations path embedded in the short-term policy rate SPF expectations (red line) shows a clear downtrend for the whole sample period, which clearly suggests the presence of a strong misalignment of expectations in recent times. Moreover, the SPF 1-year expectational term premium, identifying misaligned expectations (i.e. the wedge between the blue and red lines), increases substantially around the last three recessions. Put differently, the preliminary evidence found using SPF data supports the idea that misaligned expectations increase substantially in recessions. Interestingly, all these recessions feature high levels of the real interest rate relative to policy rate expectations, but the Great Recession and its aftermath are associated with a larger, longer rise in the expectational term premium than the other recessions.

units. Second, $log(\beta)$ is multiplied by 100 because inflation, nominal interest rates, and consumption growth rates are reported in percentage units in the SPF. Finally, the whole expression for the expectational term premium and its components shown in the graph is multiplied by 4 so that it is all measured in annualized units.

[12] These two parameter values are fairly standard and close to the estimated values reported by Smets and Wouters (2007) and Slobodyan and Wouters (2012) assuming either RE or AL.

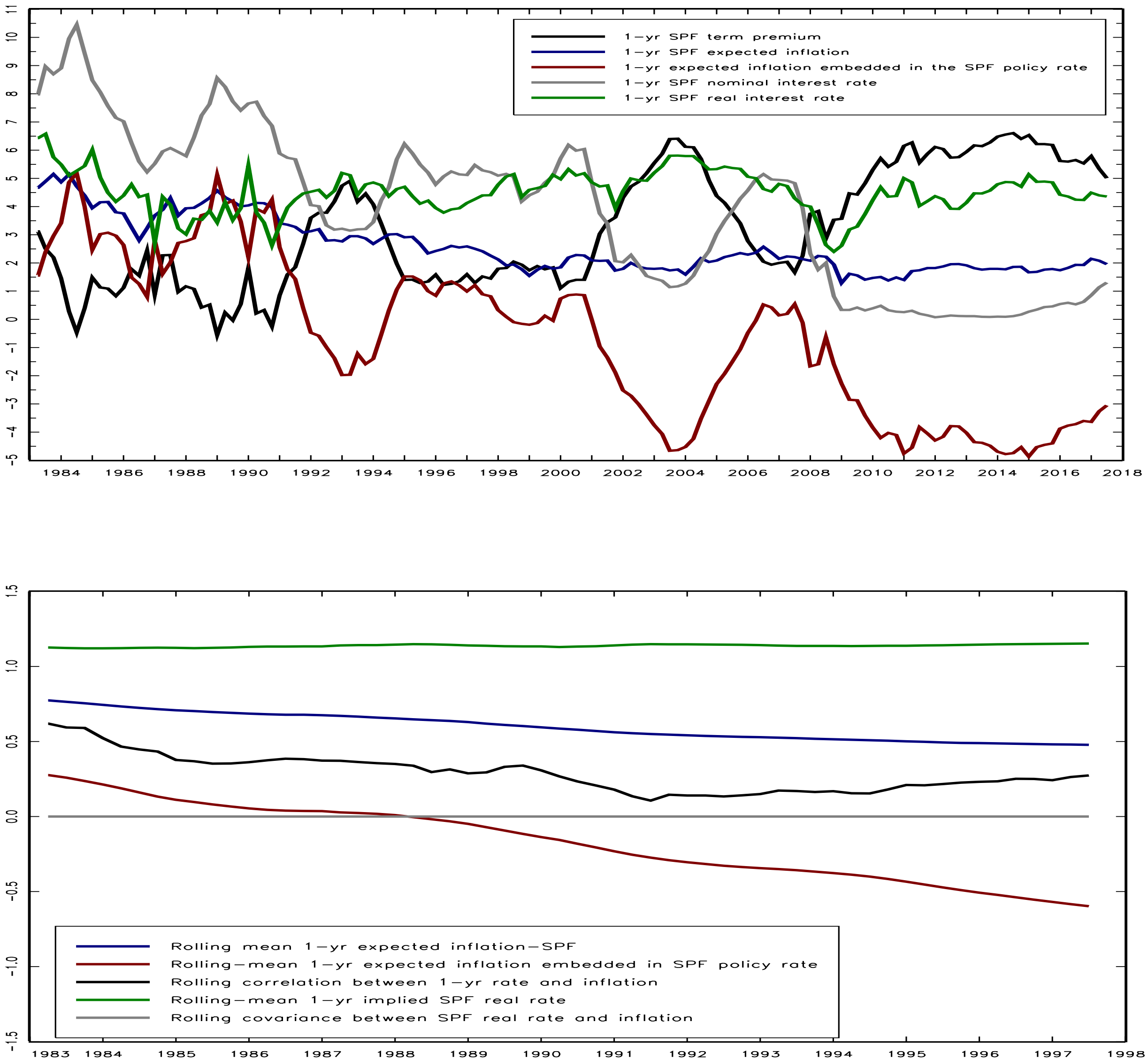

Figure 1. Decoupling of inflation expectations based on the SPF and the Gibson paradox.

Notes: All dynamic moments are computed using a 20-year rolling window. Rolling variables are measured in *quarterized* units.

All variables in the upper graph in Figure 1 show sizable short-run fluctuations which somewhat hide their low-frequency dynamics. In order to improve visualization of the latter, the bottom graph in Figure 1 shows, in *quarterized* units,[13] (i) the dynamic mean of the inflation expectations path embedded in the short-term policy rate expectations over the

[13]In contrast to the upper graph, where annualized units are used, the bottom graph uses quarterized units because they are similar in size to the dynamic correlations also plotted here.

1-year forecast horizon, $\frac{1}{4}\sum_{k=0}^{3}\left(R_{t,t+k}^{\{1\},SPF}\right) - \frac{\sigma[logC_{t+4}-logC_t]^{SPF}}{4} + 100 \times log\left(\beta\right)$ (red line); (ii) the dynamic mean of SPF inflation expectations, $\frac{1}{4}\sum_{k=1}^{4}\pi_{t,t+k}^{SPF}$, (blue line); (iii) the dynamic correlation between the 1-year yield and inflation (black line); (iv) the dynamic mean of the *ex-ante* real interest rate implied by the four-quarter-ahead SPF forecasts for consumption growth, $\frac{\sigma(logC_{t+j}-logC_t)^{SPF}}{j} - 100 \times log\left(\beta\right)$, (green line); and (v) the dynamic unconditional covariance between the implied ex-ante real interest rate and inflation using SPF forecasts for consumption growth and inflation (grey line). These dynamic moments are computed using a 20-year rolling-window (i.e. 80 quarterly observations), but the patterns of dynamic moments remain robust if, for instance, a 5-year rolling window is used. The horizontal axis in the bottom graph indicates the first quarter of the corresponding rolling window. The first three dynamic moments show a clear downtrend pattern in the bottom graph. In particular, from the early 1990s onwards all windows show a low dynamic correlation between inflation and the 1-year yield, showing evidence of the Gibson paradox at the short end of the yield curve in recent times. This finding is also aligned with the low correlation between inflation and the short-term interest rate as revealed in Cogley, Sargent, and Surico (2012) and Casares and Vázquez (2018). As discussed above, the Gibson paradox seems to be related to a disconnection between the inflation expectations of professional forecasters and those embedded in the short-term rate SPF forecasts as shown by the divergent paths of inflation expectations and those embedded in the short-term rate expectations. In contrast, the relative stability of the dynamic mean associated with the four-quarter-ahead consumption growth SPF forecasts across the rolling windows suggests that the implied dynamic mean of the ex-ante real interest rate across windows is relatively stable, which in turn suggests that the stronger downtrend in inflation expectations embedded in the short-term policy rate expectations is mainly driven by a downtrend in the short-term policy rate forecasts, $\frac{1}{4}\left(\sum_{k=0}^{3}R_{t+k}^{\{1\},SPF}\right)$, which is especially pronounced during the 2001-2004 and 2007-2009 periods.[14] Furthermore, the stability around zero of the dynamic covariance between the implied

[14]At this point, it must be clarified that the implied ex-ante real interest rate obtained from SPF data *is not* constant at all, as shown in the upper graph in Figure 1. What it is shown in the bottom graph is

1-year real interest rate and the 1-year-ahead expectations of inflation obtained from SPF forecasts provides some degree of support for the zero-covariance constraint imposed by the log-linearized model considered above.[15]

This section shows an illustration of misaligned interest rate expectations at the short-end of the yield curve using a simple consumption-based asset pricing model and SPF data. The following section considers a full-fledged DSGE model of the term structure, where model uncertainty is introduced by assuming AL with discretional beliefs. This model results in a 10-year expectational term premium capturing misaligned expectations and model uncertainty. It turns out that uncertainty, characterized here by the 10-year expectational term premium, increases greatly in recessions, as asserted by Bloom (2014).

4 A DSGE model of the term structure with adaptive learning

The model estimated builds on the Smets and Wouters (2007) model and its AL extension studied by Slobodyan and Wouters (2012a). This standard medium-scale estimated DSGE model contains both nominal and real frictions that affect the choices of households and firms. I follow Aguilar and Vázquez (2021) and Vázquez and Aguilar (2021) by extending the medium-scale DSGE model to account for the term structure of interest rates, but in contrast to their papers the long term rates here are determined by the corresponding consumption-based Euler equation instead of the EHTS. Moreover, I deviate from the monetary policy rule in the Smets and Wouters (2007) model by assuming that the monetary authorities follow a Taylor-type rule and react to *expected* inflation, output gap, output gap growth, and a

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that the rolling-mean of the implied ex-ante real interest rate across windows, capturing its low-frequency dynamics, does not show any clear trend.

[15]This unconditional covariance should be viewed as a simple proxy of the conditional covariance that shows up in equation (2). While 4-quarter-horizon forecasts of both the log of stochastic discount factor, $m_t^{\{4\}}$, and the log of compounded inflation, $log\left[\frac{1}{\prod_{k=1}^{4}(1+\pi_{t+k})}\right] = -\frac{1}{4}\left(\sum_{k=1}^{4}\pi_{t+k}\right)$, can be obtained from the SPF, the conditional covariance of these two variables is neither reported in nor can be inferred from SPF forecasts.

term spread as defined below. My purpose is to characterize the central banker and private agents as sharing a similar degree of uncertainty about the overall model economy, and about inflation in particular. As in Smets and Wouters (2007), the model contains seven structural disturbances associated with technology, demand-side, monetary policy, and price and wage markup shocks. I present the extensions of the Smets and Wouters (2007) model next. The remaining log-linearized equations of the estimated model are presented in a supplementary appendix.

4.1 The expectational term premium under AL

This section characterizes the expectational AL term premium introduced in Section 3 by augmenting the Smets and Wouters (2007) model with the term structure of interest rates and considering AL expectations with discretional beliefs. Thus, the consumption-based asset pricing equation associated with each maturity is extended to consider habit formation and labor in the description of preferences. More precisely, the standard consumption-based asset pricing equation associated with each maturity is obtained from the first-order conditions that characterize the optimal decisions of the representative consumer:

$$E_t^{AL}\left[\beta^j \frac{U_C(C_{t+j}, C_{t+j-1}^A, L_{t+j})\left((1+R_t^{\{j\}})\right)^j}{U_C(C_t, C_{t-1}^A, L_t)\prod_{k=1}^{j}(1+\pi_{t+k})}\right] = 1,\ for\ j =\ 1,\ 2,\ ...,\ n, \tag{7}$$

where E_t^{AL} stands for the AL conditional expectations operator with discretional beliefs (i.e. this AL expectations operator *does not* satisfy the law of iterated expectations),[16] L_t is labor, and the lagged *aggregate* consumption element, C_{t-1}^A, in the utility function captures the possibility of *external* habit formation. The rest of the notation is standard and identical to that used in equation (1).

Considering the multiplicative utility function assumed in the Smets and Wouters (2007)

[16] Here, we use the superscript AL, instead of D used above, to emphasize the use of a particular AL expectations operator described below.

model and further assuming a logarithmic utility function, the (linearized) consumption-based asset pricing equations can be written as[17]

$$x_t = E_t^{AL} x_{t+j} - \left(1 - \bar{h}\right) \left[j r_t^{AL\{j\}} - \sum_{k=1}^{j} E_t^{AL} \pi_{t+k} \right], \; for \; j = \; 1, \; 2, \; ..., \; n, \tag{8}$$

where the following notation is used: $x_t = c_t - \bar{h} c_{t-1}$, $\bar{h} = \frac{h}{\gamma}$. h denotes the habit formation parameter, and γ denotes the balanced-growth rate. Lower case variables denote the log-deviation of consumption, c_t, from its balanced-growth (steady-state) value or, alternatively, the deviations of the nominal yields, $r_t^{AL\{j\}}$, and the rate of inflation, π_t, from their respective steady-state values. Here, I also add the superscript "AL" to the nominal yield of the j-period maturity bond to emphasize that this yield is obtained under AL with discretional beliefs and abstracts from any Jensen's inequality terms capturing the standard interpretation of a term premium as a compensation for risk.

The expectational AL term premium, $tp_t^{AL\{j\}}$, is defined as the wedge between the nominal yield associated with the j-period maturity bond, $r_t^{AL\{j\}}$, consistent with the log-linear approximation (8) of the consumption-based asset pricing model, (7), and the nominal yield associated with the j-period maturity bond implied by the EHTS under AL with discretional beliefs, $\frac{1}{j} \sum_{k=0}^{j-1} E_t^{AL} r_{t+k}^{\{1\}}$. Formally,

$$tp_t^{AL\{j\}} = r_t^{AL\{j\}} - \frac{1}{j} \sum_{k=0}^{j-1} E_t^{AL} r_{t+k}^{\{1\}}. \tag{9}$$

As emphasized above, the expectational AL term premium, $tp_t^{AL\{j\}}$, only captures a potential misalignment between inflation expectations that shows up in the set of optimality conditions (8) and the path of inflation expectations implied by the EHTS, which are embedded in short-term interest rate expectations, once consumption growth expectations are taken into

[17]A log-utility function on consumption simplifies the estimation procedure because, among other issues, it avoids having to deal with labor expectations. This greatly reduces the number of forecasting models to be estimated under AL.

account as discussed in the previous section. In contrast to the standard interpretation of the term premium as a compensation for risk, the expectational AL term premium, $tp_t^{AL\{j\}}$, only captures a potential misalignment of inflation expectations and those inflation expectations embedded in the policy rate since the log-linear approximation (8) eliminates any Jensen's inequality terms resulting in a standard interpretation of the term premium. In short, the expectational term premium is entirely driven by misalignments in policy rate expectations implied by model uncertainty. This approach enables me to identify empirically a term premium determined by uncertainty rather than risk. This empirical strategy to distinguish uncertainty from risk is useful in the present framework because the concept of uncertainty is typically viewed as a broad one: A stand-in for a mixture of risk and uncertainty (Bloom, 2014).

As is clear in (8), consumers do not need to compute expectations for the short-term rate (or the yield consistent with the EHTS) to obtain their optimal decisions. Hence, consumers under AL with discretional beliefs are not necessarily aware of departures from the EHTS.[18] In order to compute $tp_t^{AL\{j\}}$, I consider forecasting rules (discretional beliefs) for $E_t^{AL} r_{t+k}^{\{1\}}$ similar to those used by agents to define the expectations for the forward-looking variables of the model (i.e. the combination of forecasting rules for each forecasting horizon only contains the contemporaneous and first-lag values of the forward-looking variable and the term spread, as discussed below in detail). I refer to $tp_t^{AL\{j\}}$ henceforth as the *expectational* term premium

[18] To be clear, the set of linear equations (8) should be viewed as the simple log-linear approximations of the set of (non-linear) optimality conditions (7) defined under AL with discretional beliefs used to bring the model to the data in the estimation procedure below. This view circumvents an issue raised in the AL literature. For instance, Eusepi and Preston (2018, footnote #10) argue that expected yields with multiple assets under discretional subjective beliefs in a first-order approximation may not satisfy no-arbitrage because such beliefs are inconsistent with bounded portfolio decisions. I argue that this issue is not important in this paper for several reasons. First, a reasonable AL assumption is that agents are aware of their non-rational beliefs (i.e. they know that they *do not* know the true model), so unbounded portfolio decisions cannot be optimal because departures of long-term yields from those implied by the EHTS do not guarantee riskless profits because inflation expectations embedded in short-term interest rate expectations are not necessarily aligned with the inflation expectations of agents, as suggested by the evidence discussed above. Second, optimal decisions as described in (8) do not require the computation of short-term interest rate expectations, so optimal agents under AL with discretional beliefs are not necessarily aware of departures from the EHTS. Finally, interest should not center on a first-approximation per se but on nonlinear optimality condition (7) under AL. That is, the first-order approximation of the optimality conditions must be then understood just as an approximation used to bring the model to the data.

to distinguish it from the *total* term premium, which adds an exogenous (measurement error) term to $tp_t^{AL\{j\}}$ associated with the j-period maturity yield, as also discussed below.

Long maturity term structure

The analysis of the term premium associated with the 10-year yield under the approach followed in this paper requires the use of the consumption-Euler equation associated with that yield. As implied by equation (8), the equation for the 10-year yield is given by

$$x_t = E_t^{AL} x_{t+40} - \left(1 - \bar{h}\right) \left[40 \times r_t^{AL\{40\}} - \sum_{k=1}^{40} E_t^{AL} \pi_{t+k}\right].$$

Considering a long term maturity yield such as the 10-year yield means characterizing the expectations for consumption and inflation up to a 40-quarter horizon, and the expectations for the short term rate up to the 39-quarter horizon in order to compute the associated 10-year term premium, $tp_t^{AL\{40\}}$, as shown in equation (9). In the current setup a long forecasting horizon dramatically increases the number of forecasting model parameters for consumption, inflation, and the short-term rate being estimated, potentially leading to a curse of dimensionality problem.[19] To deal with this issue, I assume the following simple recursive structure for expectations on forecast horizons beyond the four-quarter horizon:

$$\begin{cases} E_t^{AL} c_{t+j} = \mu_c E_t^{AL} c_{t+j-1}, \\ E_t^{AL} \pi_{t+j} = \mu_\pi E_t^{AL} \pi_{t+j-1}, \\ E_t^{AL} r_{t+j-1} = \mu_r E_t^{AL} r_{t+j-2}, \end{cases} \tag{10}$$

for $j > 4$. This structure builds on forecasting rules (15) and (16) described below which, among others, characterize $E_t^{AL} c_{t+4}$, $E_t^{AL} \pi_{t+4}$, and $E_t^{AL} r_{t+3}$. The parameters μ_c, μ_π, and μ_r are estimated jointly with the rest of the parameters of the model.

[19] That is, the number of learning coefficients to be estimated, described below in equations (15) and (16), increases much faster than the number of yields as longer-maturity bonds are considered.

The law of iterated expectations

To show the importance of the property of the law of iterated expectations in identifying the expectational term premium, let $\tilde{E}_t$ define an expectations operator who satisfies the law of iterated expectations (i.e. $\tilde{E}_t\left(\tilde{E}_{t+h}z_{t+j}\right) = \tilde{E}_t z_{t+j}$, for any forward-looking variable z_t and $j > h > 0$). For an expectations operator of this type, the set of optimality conditions (8) can be written as

$$x_t = \tilde{E}_t x_{t+j} - \left(1-\bar{h}\right)\left[j r_t^{\{j\}} - \sum_{k=1}^{j} \tilde{E}_t \pi_{t+k}\right], \; for \; j = \; 1, \; 2, \; ..., \; n. \tag{11}$$

In particular, for $j = 1$ equation (11) becomes[20]

$$x_t = \tilde{E}_t x_{t+1} - \left(1-\bar{h}\right)\left[r_t^{\{1\}} - \tilde{E}_t \pi_{t+1}\right].$$

Under the law of iterated expectations, this optimality condition can be iterated j-periods forward to obtain:

$$x_t = \tilde{E}_t x_{t+j} - \left(1-\bar{h}\right)\sum_{k=1}^{j} \tilde{E}_t\left[r_{t+k-1}^{\{1\}} - \pi_{t+k}\right]. \tag{12}$$

A straightforward comparison of equations (11) and (12) gives the pure EHTS:

$$r_t^{\{j\}} = \frac{1}{j}\sum_{k=0}^{j-1} \tilde{E}_t r_{t+k}^{\{1\}},$$

which implies that the expectational term premium (9) vanishes when the law of iterated expectations is satisfied, since the inflation expectations of agents and those implied by the EHTS are identical (i.e. actual inflation expectations and those embedded in the policy rate expectations are perfectly aligned in this case).

[20]Notice that the superscript AL is removed from $r_t^{\{j\}}$in equation (8) when writing (11) to distinguish the j-period nominal yield implied by AL with discretional beliefs, E_t^{AL}, from that implied by (11), where the law of iterated expectations is imposed,$\tilde{E}_t$.

An alternative AL term premium

The characterization of the expectational AL term premium suggested in this paper differs from the AL term premium considered in Vázquez and Aguilar (2021). In their paper, they focus only on the 1-period (linearized) consumption-based asset pricing equation (i.e. equation (8) for $j = 1$):

$$x_t = E_t^{AL} x_{t+1} - \left(1 - \bar{h}\right) \left[j r_t^{\{1\}} - E_t^{AL} \pi_{t+1} \right], \tag{13}$$

and assume that long term yields are determined, up to an exogenous term premium, by the EHTS. Formally, their j-period maturity yield is defined as

$$r_t^{\{j\}} = \frac{1}{j} \sum_{k=0}^{j-1} E_t^{AL} r_{t+k}^{\{1\}} + \xi_t^{\{j\}}, \tag{14}$$

where $\xi_t^{\{j\}}$ denotes their term premium defined as the wedge between the observed yield and the yield implied by the EHTS, $r_t^{AL-EH\{j\}}$. This can be interpreted as an exogenous measure of fluctuations in the risk premium (De Graeve, Emiris, and Wouters, 2009).

The following three remarks clearly distinguish the approach suggested in this paper from the one in Vázquez and Aguilar (2021). First, in Vázquez and Aguilar (2021) current consumption is determined by the *current* 1-period real interest rate and the 1-period-ahead expected consumption (as shown in equation (13)), but here the set of asset pricing equations (8) implies that current consumption (implicitly defined by the quasi consumption growth rate, x_t) depends on the *expected path* of the j-period real interest rates (i.e. $r_t^{\{j\}} - \sum_{k=1}^{j} E_t^{AL} \pi_{t+k}$) and the corresponding *expected path* of consumption (defined in $E_t^{AL} x_{t+j}$). Second, by imposing the EHTS to determine long-term yields, Vázquez and Aguilar (2021) ignore the presence of misaligned expectations when modeling the term premium under AL with discretional beliefs. Finally, the (*endogenous*) expectational AL term premium described in (9) should also be distinguished from the *exogenous* AL term premium, $\xi_t^{\{j\}}$, studied in Vázquez and Aguilar

(2021). As is standard in empirical applications of term structure models (e.g. De Graeve, Emiris, and Wouters, 2009), Vázquez and Aguilar (2021) augment the EHTS with an exogenous term premium shock, as in (14), which is viewed as an exogenous *convenience yield term* (see, among others, Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood, Hanson, and Stein, 2015; Del Negro, Giannone, Giannoni, and Tambalotti, 2017) related to the safety and liquidity features of (US) government bonds relative to assets with the same payoff, but without such singular properties. As an alternative to their exogenous term premium specification, this paper provides a characterization of an (endogenous) expectational term premium under AL with discretional beliefs that does not impose the EHTS in determining long-term yields.

4.2 Adaptive learning

This paper departs from the RE assumption by considering a type of behavioral AL which places lack of knowledge about the economic environment at center stage in the characterization of the term premium. Under AL, agents do not know the structure of the economy, so they face a first-order uncertainty. That is, RE agents are able to identify all sources of uncertainty, but AL agents have to learn—in general—about how the economy behaves, and in particular about the alternative sources of fundamental model uncertainty from the time series that they observe when they are forming their expectations in an imperfect information setup.

By considering small forecasting models (e.g. Adam (2005), Branch and Evans (2006), Slobodyan and Wouters (2012a,b), Rychalovska, Slobodyan, and Wouters (2016), and Vázquez and Aguilar (2021)), this paper deviates from the minimum state variable (MSV) AL approach followed by Eusepi and Preston (2011) and others (Orphanides and Williams, 2005; Milani, 2007, 2008, 2011; Sinha, 2015, 2016), where agents' expectations are assumed to be based on a (linear) function of the state variables of the model. In contrast, small forecasting models assume that agents generate their expectations based on the information provided by endogenous variables, such as those appearing in the optimality conditions of a DSGE model.[21]

Next, I provide a brief description of how AL expectation formation works.[22] A DSGE model can be represented in matrix form as follows:

$$A_0 \begin{bmatrix} \mathbb{Z}_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} \mathbb{Z}_t \\ w_t \end{bmatrix} + A_2 E_t^{AL} \mathbb{Z}_{t+j} + B_0 \epsilon_t = 0,$$

where $\mathbb{Z}_t$ is the vector of endogenous variables at time t, $E_t^{AL}\mathbb{Z}_{t+j}$ contains multi-period-ahead expectations (i.e. it includes longer-dated forecasts of consumption, inflation, and the short-term interest rate), and w_t is a vector including seven exogenous shocks and the lagged innovations, ϵ_{t-1}, of the price- and wage-markup shocks since they are modeled as ARMA(1, 1) processes.

Agents are assumed to use small forecasting models (i.e. their *perceived* law of motion processes) defined as follows:

$$Z_{t+j} = X_t \beta_{t-1}^{\{j\}} + u_{t+j},\ for\ j = \ 1,\ 2,\ ...,\ n,$$

where Z_t is the vector containing the forward-looking variables of the model (i.e. Z_t is included in $\mathbb{Z}_t$), X_t is the matrix of regressors, $\beta_t^{\{j\}}$ is the vector of updating parameters, which includes an intercept, and u_t is a vector of errors. Those errors are linear combinations of the true model innovations. Therefore, the variance-covariance matrices, $\Sigma = E[u_{t+j}u_{t+j}^T]$, are non-diagonal. Agents are further assumed to use simple econometric tools under AL. More precisely, they use a linear least squares projection in which the parameters are updated to

[21] As argued in Aguilar and Vázquez (2021), small forecasting models can be thought of as a more appealing approach to AL than MSV on several grounds. Small forecasting models are robust to alternative models characterized by different MSV sets. This is an important feature because one of the main motivations for moving from the RE assumption to some sort of AL is that agents do not really know what the true model is. Consequently, they are uncertain about the actual MSV set. Taking into account the limited information scenario faced by agents in reality seems to be an important feature in estimating the term premium: That premium is characterized by expectation misalignment, which is driven by model uncertainty and thus fully abstracts from the standard view of a term premium as a compensation for risk as emphasized above.

[22] For a detailed explanation see Slobodyan and Wouters (2012a).

form their expectations for each forward-looking variable: $E_t^{AL} Z_{t+j} = X_t \beta_{t-1}^{\{j\}}$. The updating parameter vector, β_t, which results from stacking all the vectors $\beta_t^{\{j\}}$, is further assumed to follow an autoregressive process where agents' beliefs are updated through a Kalman filter as described below. This expectation updating process is represented as in Slobodyan and Wouters (2012a) by the equation $\beta_t - \bar{\beta} = F(\beta_{t-1} - \bar{\beta}) + v_t$, where F is a diagonal matrix with the learning parameter $\mid \rho \mid \leq 1$ on the main diagonal and v_t are i.i.d. errors with variance-covariance matrix V. This standard AL approach assumes that agents do not take into account the fact that their belief coefficients will be revised in the future—e.g. Sinha (2016), and Eusepi and Preston (2011, 2018).[23]

Once the expectations of the forward-looking variables,$E_t^{AL} Z_{t+j}$, are computed they are plugged into the matrix representation of the DSGE model to obtain a backward-looking representation of the model as follows:

$$\begin{bmatrix} \mathbb{Z}_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} \mathbb{Z}_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t,$$

where the time-varying matrices μ_t, T_t and R_t are nonlinear functions of structural parameters (entering into matrices A_0, A_1, A_2 and B_0) together with the learning coefficients, β_t. This representation of the model is called the *actual* law of motion.

The Kalman-filter updating and transition equations for the belief coefficients and their corresponding covariance matrix are given by

$$\beta_{t|t} = \beta_{t|t-1} + R_{t|t-1} X_{t-1} \Big[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \Big]^{-1} \Big(Z_t - X_{t-1} \beta_{t|t-1} \Big),$$

where $(\beta_{t+1|t} - \bar{\beta}) = F(\beta_{t|t} - \bar{\beta})$, $\beta_{t|t-1}$ is the estimate of β_t using the information up to time $t-1$ (but also considering the autoregressive process followed by β_t), and $R_{t|t-1}$ is the mean

[23]This assumption can be rationalized by using an anticipated utility approach such as that put forward in Kreps (1998) and Sargent (1999). Such an approach assumes that agents do not take into account future updates of beliefs when making current decisions but are otherwise fully optimal. This is in contrast to the Bayesian belief approach followed by Adam, Marcet, and Beutel (2017), which takes belief updates into account.

squared error associated with $\beta_{t|t-1}$. Therefore, the updated learning vector $\beta_{t|t}$ is equal to the previous one, $\beta_{t|t-1}$, plus a correction term that depends on the previous forecast error, $\left(Z_t - X_{t-1}\beta_{t|t-1}\right)$. The mean squared error, $R_{t|t}$, associated with this updated estimate is given by

$$R_{t|t} = R_{t|t-1} - R_{t|t-1}X_{t-1}\left[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1}\right]^{-1} X_{t-1}^T R_{t|t-1}^{-1},$$

with $R_{t+1|t} = FR_{t|t}F^T + V$.

The initialization of this Kalman filter for the belief coefficients requires that $\beta_{1|0} = \overline{\beta}$, $R_{1|0}$, Σ, and V be specified. I follow Slobodyan and Wouters (2012a), where all these expressions are derived from the correlations between the model variables implied by the RE equilibrium computed at the corresponding structural parameter vector.

Forecasting models based on small information sets

I consider that agents combine two alternative forecasting models at the same time, track their forecasting performance, and use a variant of the Bayesian model averaging method to generate an aggregate forecast from the alternative forecasting models that is used to characterize their decisions.[24] In the first forecasting model I follow Slobodyan and Wouters (2012a) by assuming a perceived law of motion (PLM) featuring a second-order autoregressive process for each one-period-ahead conditional expectation that shows up in the DSGE model analyzed. I also consider the contemporaneous and the first lag for the PLM that characterizes the remaining j-period-ahead conditional AL expectations. Formally,

$$m_1: \quad E_t^{AL} Z_{t+j} = \theta_{1,Z,t-1}^{\{j\}} + \beta_{1,Z,t-1}^{\{j\}} Z_t + \beta_{2,Z,t-1}^{\{j\}} Z_{t-1}. \tag{15}$$

[24] More precisely, for each forecasting model m_i the agents track the value of

$$B_{i,t} = t \cdot log\left(det\left(\frac{1}{t}\sum_{i=1}^{t} u_i u_i^T\right)\right) + \kappa_i \cdot log(t),$$

where κ_i is the number of degrees of freedom in the forecasting model m_i and u_i is the i-th model forecasting error. As pointed out in Slobodyan and Wouters (2008), this expression is a generalization of the sum of squared errors adjusted for degrees of freedom using the Bayesian information criterion penalty. Thus, given values of $B_{i,t}$, the weight of a model i at time t is proportional to $exp\left(-\frac{1}{2}B_{i,t}\right)$.

The second forecasting model follows Aguilar and Vázquez (2021) and Vázquez and Aguilar (2021) by also considering the 1-year term spread—$sp_t^{\{4\}} = r_t^{\{4\}} - r_t$. Formally,

$$m_2: \quad E_t^{AL} Z_{t+j} = \theta_{2,Z,t-1}^{\{j\}} + \gamma_{Z,t-1}^{\{j\}} sp_t^{\{4\}}. \tag{16}$$

A comparison of forecasting models (15) and (16) shows that the latter is simpler and contains term spread information, which is observed in real time and thus entertains a realistic feature in the learning process, while the former is well suited to characterizing expectations in macroeconomic scenarios featuring large persistence. The use of the PLM (16) is further motivated by the ability of term spreads to predict inflation (Mishkin, 1990) and real economic activity (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997). It turns out that the estimation results show that the forecasting model (16) fully characterizes the learning process since 1988. This date roughly coincides with the end of the disinflation process in the US that started in the early 1980s, and the start of a period featuring low inflation persistence and the resurgence of the Gibson paradox as documented in Cogley, Sargent, and Surico (2012) and Casares and Vázquez (2018). This fall in inflation persistence may explain why the forecasting model (15) plays no role in describing expectation dynamics since 1988 as also discussed below.

In line with the PLM (15)-(16), discretional beliefs are considered. Thus, each expectational horizon is estimated separately, so they do not have to be consistent with each other. As discussed above, this AL approach based on direct multi-step forecasting overcomes the potential weakness of alternative AL models relying on iterated forecasts obtained from a misspecified forecasting model because misspecification errors can be compounded with the forecast horizon.[25,26] Through the time-varying learning parameters, the AL approach intro-

[25] Preliminary estimation attempts show that the combination of the two alternative small forecasting models helps to identify the learning parameter, ρ, in the approach based on the set of optimality conditions followed in this paper.

[26] This multi-step forecasting approach to AL is in clear contrast to the maintained beliefs hypothesis suggested in Preston (2005) (and also followed in Eusepi and Preston (2011) and Sinha (2015, 2016)), which not only imposes an infinite forecast horizon but also considers iterated forecasts used under the MSV approach.

duces some non-linear features that help somewhat to overcome the log-linear approximation typically used in DSGE models. As emphasized in Aguilar and Vázquez (2021), this feature improves model fit by capturing low frequency patterns in the data captured by the time-varying intercepts, $\theta^{\{j\}}_{i,Z,t-1}$, in equations (15) and (16). Moreover, the combination of different small forecasting models in the AL model adds flexibility, and somewhat resembles how SPF panelists forecast, as discussed above.

PLM disciplined by the Survey of Professional Forecasters

Readers might wonder whether the AL approach considered here is too flexible in adding degrees of freedom. To cope with this issue, and as a way of disciplining the expectations for the short-term interest rate implied by the EHTS, which are crucial in defining the term premium, I assume that the deviation over the current and the next three quarters in the average of the short-term interest rate AL model expectations $\left(\frac{1}{4}\sum_{k=0}^{3} E_t^{AL} r_{t+k}^{\{1\}}\right)$ from their observable counterparts reported in the SPF, denoted by $\epsilon^{\{4\}}_{SPF,r,t}$, follows a first-autoregressive process.[27] Notice that the 1-year yield implied by the EHTS is given by $\frac{1}{4}\sum_{k=0}^{3} E_t^{AL} r_{t+k}^{\{1\}}$. Therefore, using the observable counterpart of $\frac{1}{4}\sum_{k=0}^{3} r_{t+k}^{\{1\},SPF}$ in the SPF helps to discipline the estimated 1-year expectational AL term premium.

Readers may also wonder why I only discipline the expectations of the short-term interest rate. There are three main reasons: First, unlike expectations for other variables (consumption and inflation), short-term interest rate expectations do not fall within the set of optimality conditions under AL with discretional beliefs as described by equation (8) and, thus, they are not disciplined by model's dynamics. Therefore, it is important to discipline

[27] Several authors suggest that it may be appropriate to allow for the possibility of serial correlation in noise to accommodate a realistic departure of market forecasts from survey forecasts. In particular, Kim and Orphanides (2012) argue that survey forecasts only serve as a noisy source of information on expectations since surveys may not always represent the expectations of market participants. Cohen, Hördahl, and Xia (2018) argue that forecasters may compete for business or for influence through their calls. Moreover, surveys are typically computed as equal-weighted averages of responses, whereas market yields are determined by the expectations of the marginal investor (Li, Meldrum, and Rodriguez, 2017). Furthermore, a few participants may have a huge impact on the market and they may have an informational advantage over professional forecasters.

those expectations critically defining the EHTS, and hence the expectational AL term premium with discretional beliefs in (9) with survey-based data on expectations. Second, as shown in Vázquez and Aguilar (2021), the need to discipline AL expectations is greatly reduced by including term structure information in the forecasting models as I also do in this paper, as discussed above. Finally, the inclusion of SPF forecasts to discipline consumption and inflation expectations would further increase the number of observables considered in measurement equation (18) below, which is already large. In spite of these arguments, a sensitivity analysis is carried out below by also including the average of the SPF inflation forecasts over the next four quarters, $\frac{1}{4}\sum_{k=1}^{4}\pi_{t+k}^{SPF}$, in the set of observables in order to discipline further the learning coefficients featuring the AL forecasting model for inflation expectations. As shown below, the dynamics of the estimated AL term premium are not altered when the set of observables is augmented with SPF inflation forecasts.

4.3 A forward-looking policy rule

As in Slobodyan and Wouters (2012a), I deviate from the monetary policy rule in the Smets and Wouters (2007) model by assuming that the monetary authorities follow a Taylor-type rule, reacting to inflation, output gap, and output gap growth, where the output gap is defined as the deviation of output from its underlying neutral productivity process. The monetary policy rule assumed in Slobodyan and Wouters (2012a) is also slightly modified to include (i) the 1-year term spread, $sp_t^{\{4\}} = r_t^{\{4\}} - r_t^{\{1\}}$, as in Vázquez and Aguilar (2021);[28] and (ii) AL inflation expectations instead of contemporaneous inflation. This forward-looking rule enables *informational symmetry* to be maintained between the private sector and the central bank (i.e. the inflation expectations of the two types of agent coincide). Formally,

[28] McCallum (1994) and Vázquez, María-Dolores, and Londoño (2013) are early papers that discuss the role of term spreads as simple predictors of future macroeconomic conditions in the characterization of monetary policy. This assumption is also in line with Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) who augment the standard Taylor rule to include the excess return on a longer-term bond.

the policy rule is given by

$$r_t^{\{1\}} = \rho_r r_{t-1}^{\{1\}} + (1-\rho_r)\left[r_\pi E_t^{AL}\pi_{t+1} + r_y \hat{y}_t\right] + r_{\Delta y}\Delta\hat{y}_t + r_{sp} sp_t^{\{4\}} + \varepsilon_t^r, \tag{17}$$

where the output gap is defined as $\hat{y}_t = y_t - \Phi\varepsilon_t^a$ (i.e. as the deviation of output from its underlying neutral productivity process) and the policy shock, ε_t^r , is assumed to follow an AR(1) process with a persistence parameter denoted by ρ_R, as in the Smets and Wouters (2007) model.

5 Estimation results

This section starts with a description of the data and the estimation approach, then goes on to discuss the model fit and estimation results.

5.1 Data and estimation approach

The AL-DSGE model with multiple asset price equations suggested here and the AL model in Vázquez and Aguilar (2021) (henceforth, VA-AL specification) are estimated using US quarterly data for 1983:2-2017:3. The set of observable variables is identical to the one considered by Slobodyan and Wouters (2012a) (i.e. the quarterly series of the inflation rate, the federal funds rate, the log of hours worked, and the quarterly log differences of real consumption, real investment, real wages and real GDP) with the addition of the 1- and 10-year, zero-coupon Treasury yields and the average of the SPF (nowcast) forecasts of the three-month TB rate from 0- to 3-quarter horizons. GDP, consumption, investment, and hours worked are measured in per-working age population terms. The sample period includes the Great Recession, so I consider the shadow rate suggested by Wu and Xia (2016) to deal with the zero-lower-bound issue that affects it. The shadow rate is the same as the federal funds rate when the zero-lower-bound is not binding, but it is negative to account for unconventional policy tools implemented when the federal funds rate is close to the zero lower bound (roughly from 2009:1 to 2015:4). Recent papers (e.g. Wu and Zhang, 2019; Mouabbi and Sahuc, 2019; and Aguirre and Vázquez, 2020) use the shadow rate as a replacement for the federal funds rate in New-Keynesian frameworks.

The set of measurement equations is

$$\begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ dlP_t \\ lHours_t \\ FEDFUNDS_t \\ 1-year\,TB\,yield_t \\ 10-year\,TB\,yield_t \\ r_t^{\{4\},SPF} \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\pi} \\ \bar{l} \\ \overline{r} \\ \overline{r} \\ \overline{r} \\ \overline{r}^{SPF} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ \frac{1}{4}\sum_{k=0}^{3} E_t^{AL} r_{t+k}^{\{1\}} + tp_t^{AL\{4\}} + \epsilon_{r,t}^{\{4\}} \\ \frac{1}{40}\sum_{k=0}^{39} E_t^{AL} r_{t+k}^{\{1\}} + tp_t^{AL\{40\}} + \epsilon_{r,t}^{\{40\}} \\ \frac{1}{4}\sum_{k=0}^{3} E_t^{AL} r_{t+k}^{\{1\}} + \epsilon_{r,t}^{\{4\},SPF} \end{bmatrix}, \qquad (18)$$

where l and dl denote the log and the log difference, respectively. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate for real GDP, real consumption, real investment, and real wages. These are the variables that feature a long-run trend. $\bar{l}$, $\overline{\pi}$ and $\overline{r}$ are the steady-state levels of hours worked, inflation, and the short-term interest rate. The superscripts SPF and $\{4\}$ in the last row of the measurement equation denote actual average forecasts from the SPF and the corresponding forecast horizon, respectively. Hence, the measurement equation involves 10 observable variables and three measurement errors, $\epsilon_{r,t}^{\{4\}}$, $\epsilon_{r,t}^{\{40\}}$, and $\epsilon_{r,t}^{\{4\},SPF}$ associated with the 1-year and 10-year yields, and the average of the SPF forecasts of the short-term nominal rate over the current and next three quarters, respectively.[29] I allow for persistence in the measurement errors associated with the 1-year and 10-year yields

[29] Together with the seven structural shocks, the inclusion of these three measurement errors implies that the sum of structural shocks and measurement errors is (at least) equal to the number of observables used in the estimation procedure, which overcomes the *stochastic singularity* that would otherwise show up, making estimation impossible.

($\epsilon_{r,t}^{\{4\}}$and $\epsilon_{r,t}^{\{40\}}$, respectively). Importantly, these measurement errors capture any other components determining the actual yields, such as the component of the term premium typically viewed as the additional return for the risk associated with a long-term bond, which is totally ignored when (linearized) consumption-based asset pricing equations (8) are considered. According to the penultimate equation of (18), the total 10-year AL term premium (i.e. the wedge between the 10-year yield and the 10-yield implied by the EHTS, $\overline{r}+\frac{1}{40}\sum_{k=0}^{39}E_t^{AL}r_{t+k}^{\{1\}}$) is thus defined as the sum of two elements: (i) The expectational AL term premium, $tp_t^{AL\{40\}}$; and (ii) the measurement error, $\epsilon_{r,t}^{\{40\}}$.[30] In a sensitivity analysis reported below, the measurement equation (18) is augmented by also including the average of the SPF forecasts of inflation over the next four quarters in order to assess the robustness of the estimated expectational term premium measure. Below, I also assess the robustness of the expectational AL term premium by extending the sample period to the last quarter of 2019.

The estimation uses the Bayesian estimation procedure commonly used in the related DSGE literature. First, the log posterior function is maximized by combining prior information on the parameters with the likelihood of the data. The prior assumptions are exactly the same as in Slobodyan and Wouters (2012a). Moreover, I consider rather loose priors for the parameters that characterize both bond term premium dynamics and the measurement errors considered. The second step implements the Metropolis-Hastings algorithm, which runs a long sequence of a million draws of all the possible realizations for each parameter to obtain its posterior distribution.

As stated above, in addition to the calibrated parameters considered in Slobodyan and Wouters (2012a), I set the relative risk aversion parameter to one (that is, a log-utility function on consumption is assumed). This restriction is imposed for several reasons. First, it avoids having to deal with hours-worked expectations, which reduces the number of fore-

[30] I do not further decompose the expectational AL term premium into real and inflation term premium components in this paper. Such a decomposition would require real term structure data to be considered in addition to the large number of observed variables already used in the paper. Moreover, the use of real term structure data is somewhat problematic because real zero-coupon yields can be obtained only from 1999 onwards. Hördahl and Tristani (2014) overcome this problem by considering real yields as unobservable variables before 1999 in the estimation procedure of their macroeconometric-term-structure model.

casting models to be estimated. Second, it imposes a parameter value for the relative risk aversion that is much closer to a standard parameterization of the utility function which, in addition, mutes the possibility of the expectational term premium picking up any component of the term premium consistent with the standard view of a compensation for the risk associated with long-term bonds since a very large value of the risk aversion parameter is needed to generate this type of standard term premium— see e.g. Rudebusch and Swanson (2008, 2012); and Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012)— with high-order approximations of the non-linear consumption-based asset pricing equation as discussed above.

5.2 Model fit

A comparison of the (log) marginal likelihood values associated with the AL model with multiple asset price equations suggested here and the VA-AL specification suggested in Vázquez and Aguilar (2021) results in a substantial improvement in model fit of $[-371.03 - (-445.73) =]$ 74.70 log-points in favor of the former, as shown in the bottom line of Table 1 below.

Figure 2 shows the plots for the observable variables used in the estimation procedure (solid blue line) together with the 1-quarter-ahead forecasts associated with the baseline AL model suggested in this paper (dotted red line) and the VA-AL specification (dashed green line). In both cases the AL model fits the macroeconomic time series, the 1-year and 10-year zero-coupon yields, and the short-term rate SPF forecasts reasonably well. In particular, the root mean squared error (RMSE) statistics based on in-sample forecasts, shown in each graph of Figure 2 for the two models, indicate that the fit is very good for all the nominal variables used in the estimation procedure. The baseline specification also greatly improves the fit of investment and real wage growth rates, while the VA-AL specification does a better job in fitting consumption growth. Furthermore, the RMSE-statistics for the two specifications are also in line for the alternative variables with those reported in Slobodyan and Wouters

(2012a), who estimate a similar AL-DSGE model but do not consider the term structure block of the model (or term structure information in the forecasting models) considered in the baseline and the VA-AL specifications.

A supplementary appendix analyzes the performance of the baseline- and VA-AL specifications in reproducing selected second-moment statistics obtained from actual data. The stochastic simulation of the two model specifications shows that they both provide reasonable characterizations of the macroeconomy and the variables that describe the term structure of interest rates (i.e. the 1-year and 10-year yields and the 1-year yield implied by the EH of the term structure based on the SPF expectations for the short-term interest rate, $r_t^{SPF\{4\}}$). These features mean that the two AL specifications studied are appropriate tools for analyzing the properties of the term premiums associated with them.

The estimation results also show that the weight associated with each of the two forecasting models is 0.5 for around the first five years of the sample (roughly until 1988), while forecasting model (16) (based only on term spread information) fully characterizes the expectations of forward-looking variables for the rest of the sample. This finding highlights the major importance of term structure information in characterizing learning dynamics. Moreover, the fall in inflation persistence reported in Cogley, Sargent and Surico (2012) and Casares and Vázquez (2018) may explain why the forecasting model (15), which is prone to capture persistent dynamics, plays no role in describing expectation dynamics after 1988.

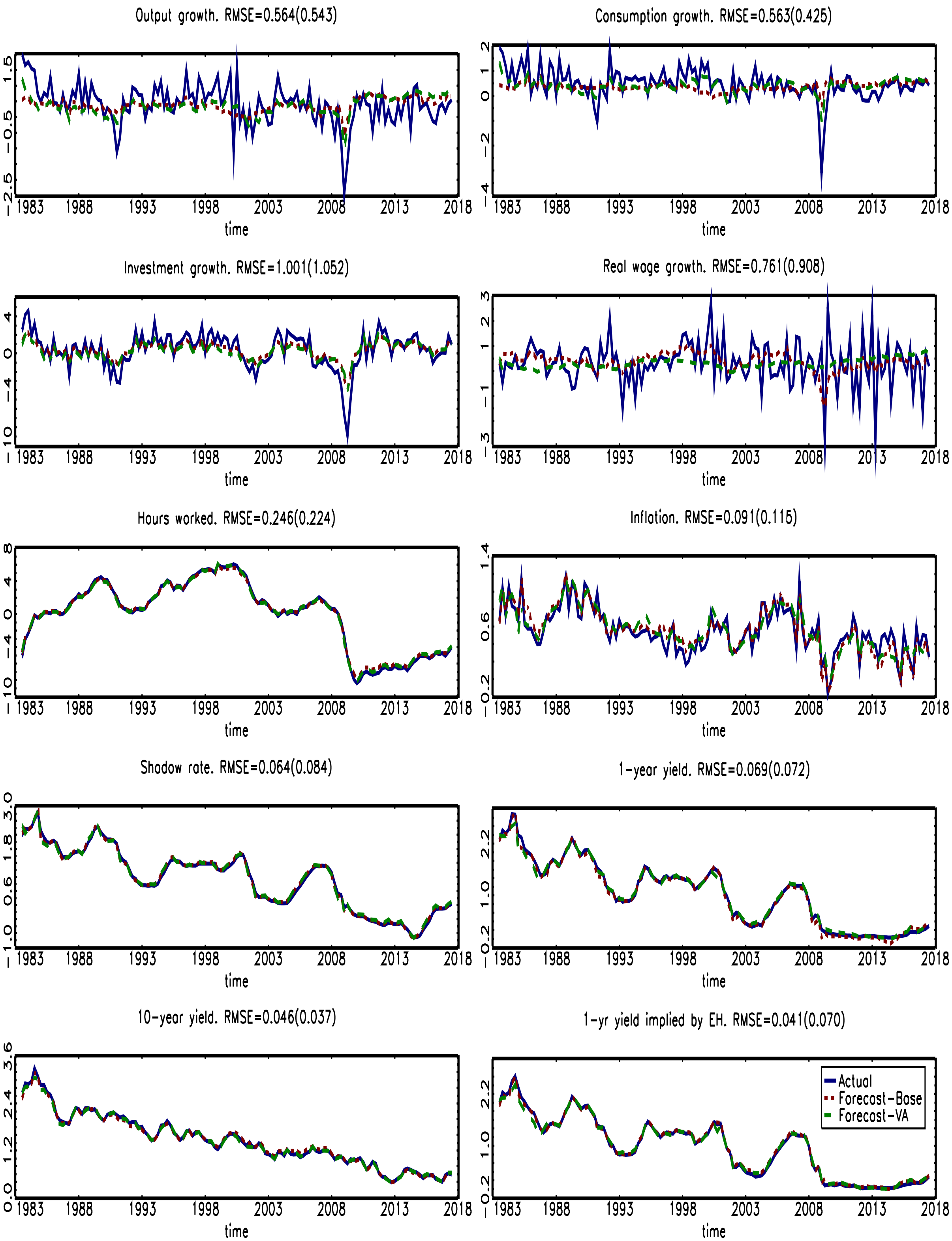

Figure 2. Historical and one-quarter-ahead forecast time series yields.

Note: Each graph shows the RMSE-statistics for the baseline AL model and the VA-AL specification (in parenthesis).

5.3 Posterior estimates

Table 1 shows the estimation results for a selected group of parameters featuring both endogenous and exogenous persistence for the baseline- and VA-AL models. The estimate of the learning parameter ρ (around 0.94) is similar for the two specifications.[31] These estimates of the learning parameter are also similar to those reported in Slobodyan and Wouters (2012a) and Vázquez and Aguilar (2021) using the data up to the end of the Great Moderation period.

An analysis of the differences in posterior estimates obtained from the two specifications reveals that (i) habit formation is much lower in the baseline-AL specification than in the VA-AL one (0.26 versus 0.71), but the opposite occurs for the capital utilization adjusting cost parameter (0.75 versus 0.03); (ii) Calvo wage probability is also much lower for the baseline model (0.24 versus 0.59); (iii) the moving-average coefficients of markup shocks are lower in the baseline model, especially for markup wage shocks; and (iv) regarding policy rule parameters, the baseline AL model results in higher persistence and higher interest rate responses to both inflation and output gap than the VA-AL model, while the term spread coefficient, r_{sp}, takes a similar value for the two specifications and is in line with those reported in Aguilar and Vázquez (2021) and Vázquez and Aguilar (2021) under AL, thus showing a robust response of the policy rate to the 1-year term spread.

[31] A supplementary appendix shows the full set of parameter estimates for the two specifications.

Table 1. Selected parameter estimates (Sample period: 1983:2-2017:3)

Parameters associated with real rigidities		*Parameters associated with markups*	
habit formation (h)	0.26/0.71	mark-up price AR coef. (ρ_p)	0.98/0.95
	(0.23,0.29)		(0.96,0.99)
cost of adjusting capital (φ)	2.61/2.65	mark-up wage AR coef. (ρ_w)	0.96/0.96
	(2.33,2.79)		(0.94,0.98)
capital utilization adjusting cost (ψ)	0.75/0.03	mark-up price MA coef. (μ_p)	0.47/0.58
	(0.69,0.81)		(0.41,0.52)
Parameters associated with nominal rigidities		mark-up wage MA coef. $(\mu_w$	0.17/0.50
Calvo price probability (ξ_p)	0.66/0.63		(0.11,0.21)
	(0.61,0.71)	*Policy rule parameters*	
Calvo wage probability (ξ_w)	0.24/0.59	inertia (ρ_r)	0.97/0.86
	(0.21,0.27)		(0.96,0.98)
price indexation (ι_p)	0.17/0.14	inflation (r_π)	1.56/1.37
	(0.11,0.23)		(1.45,1.64)
wage indexation (ι_w)	0.51/0.48	output gap (r_y)	0.16/0.00
	(0.45,0.58)		(0.13,0.20)
Learning parameter (ρ)	0.93/0.95	output gap growth $(r_{\Delta y})$	0.03/0.01
	(0.92,0.94)		(0.02,0.03)
log data density	-371.03/-445.73	term spread (r_{sp})	0.18/0.17
			(0.15,0.21)

Note: Each cell shows the posterior parameters estimates for the baseline- and VA-AL specifications, respectively. Parameter notation and credible sets for the baseline specification are in parentheses. The credible sets for the VA-AL specification are available in the supplementary appendix.

6 AL term premium features

This section analyzes the AL term premium features associated with the 10-year yield. To that end, I compare the annualized estimated AL term premiums with those estimated from the no-arbitrage affine models of Kim and Wright (2005) and Adrian, Crump and Moench (2013). Figure 3 shows the estimated term premiums associated with the 10-year bond yield from the expectational AL model suggested in this paper (black line) together with the five-factor no-arbitrage affine model of Adrian, Crump and Moench (2013) (blue line) (henceforth called the ACM term premium), and the three-factor arbitrage-free term structure model estimated by Kim and Wright (2005) (purple line) (henceforth called the KW) for the sample period 1990:3-2017:3.[32] In order to assess the robustness of the results, Figure 3 also plots the 10-year bond yield from the expectational AL model when the average of the SPF forecasts of inflation over the next four quarters is included in the set of observables. Several conclusions can be drawn from this figure. First, the two expectational AL term premium measures have very similar patterns. The two expectational AL term premiums are sizable and show wide fluctuations as the alternative measures. Second, there are major (and persistent) departures (around 200 basis points or even higher) between alternative measures of the term premium. Importantly, a strong comovement between the expectational AL measures and the ACM term premiums is observed since 2001. Thus, large increases can be seen in the expectational term premium measures and the (five latent factors) ACM measure after the 2001 and 2007-2008 crisis, which contrast with the mild increase in the (three latent factor) KW measure. This finding suggests that the two additional latent factors considered in the estimation of the ACM measure might well be associated with the increasing uncertainty generated by these recessions, which is well captured by the estimated expectational AL term premium. This finding is also aligned with the idea that "... *uncertainty rises in recessions*", as postulated by Bloom (2014). Finally, the rapid fall of the expectational term premium since 2011 suggests

[32]Notice that the first observation (1990:3) in this comparison analysis of alternative term premiums is different from the first observation in the estimated sample period (1983:2). This is because the KW term premium time series is available only from 1990:3 onwards.

that the expectation misalignment resulting from uncertainty captured by this term premium measure has been losing importance since the end of the Great Recession.

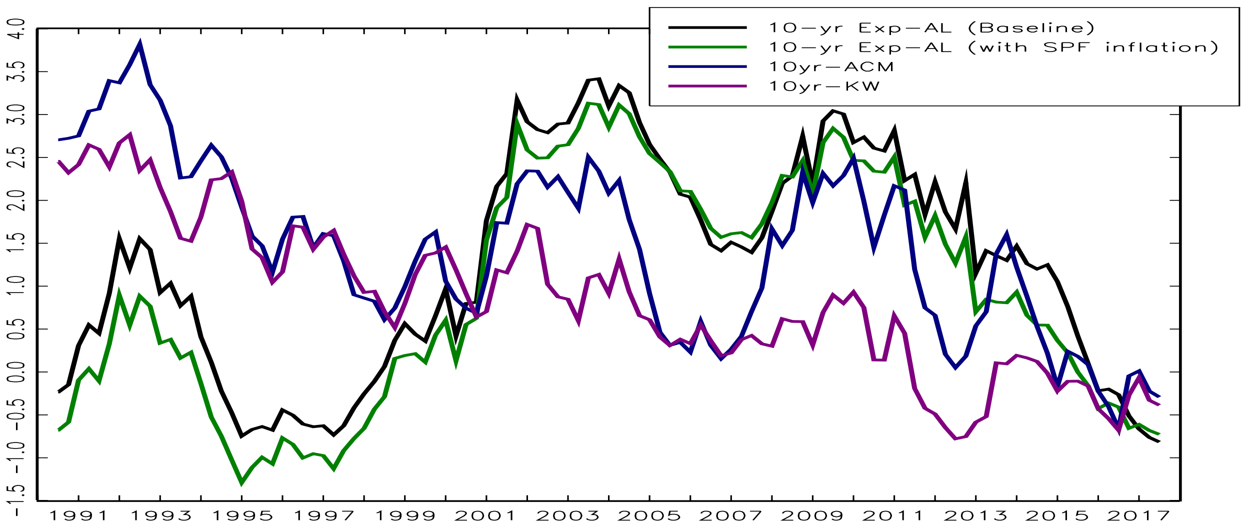

Figure 3. Annualized 10-year term premiums.

Notes: The two annualized *expectational* AL 10-year term premium measures reported in this figure are computed as $4 \times tp_t^{AL\{40\}}$.

Figure 4 illustrates the robustness of the expectational AL term premium to an extension of the estimation sample to the last quarter of 2019, excluding the Covid-19 pandemic period and its aftermath. For this extended sample, the estimated expectational AL moves a little closer to the ACM term premium since 2001 than for the shorter sample period.

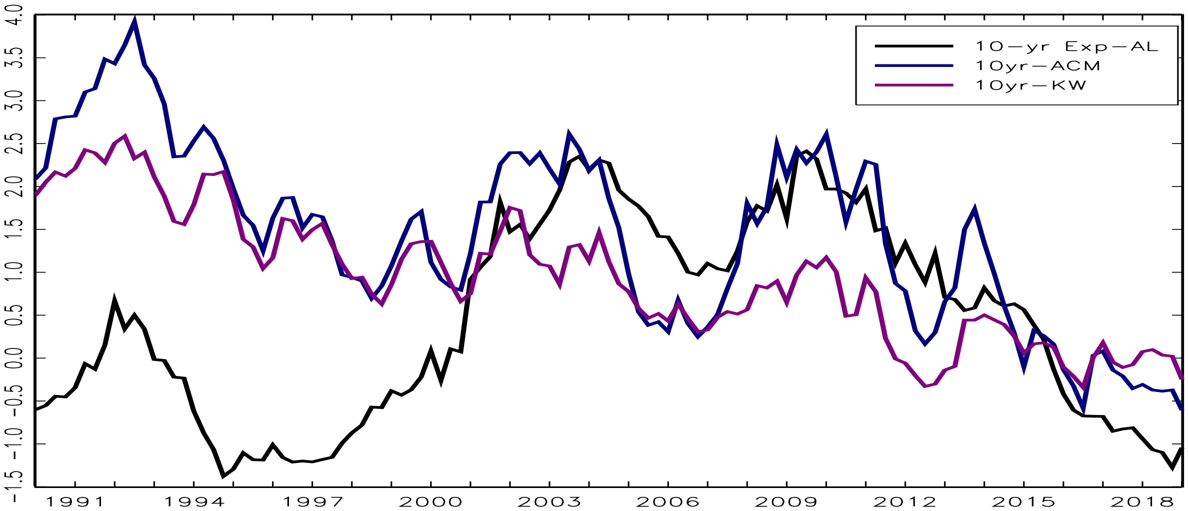

Figure 4. Annualized 10-year term premiums for an extended sample.

Notes: The *expectational* AL 10-year term premium is obtained by estimating the model for the extended period 1983:2-2019:4.

Table 2 goes beyond the visual presentation provided by Figures 3-4 and shows statistics (standard deviation, first-order autocorrelation, and contemporaneous cross-correlations) that point to further similarities and differences between the alternative term premiums computed for the sample period 1990:3-2017:3. In addition to the term premiums displayed in Figure 3, this table also considers three other AL term premium measures: (i) The estimated exogenous measure of the 10-year AL term premium obtained with the baseline set of observables and computed as $4 \times \epsilon_{r,t}^{\{40\}}$; (ii) a total measure of the 10-year term premium, which adds this exogenous measure to the baseline expectational AL measure; and (iii) the exogenous 10-year term premium based on the AL model of Vázquez and Aguilar (2021). The standard deviations of the total and expectational term premiums associated with the AL-DSGE model suggested here can be seen to be higher than the standard deviations of the other term premiums except for that of the exogenous AL term premium. Moreover, the first-order correlation statistic indicates that AL induces a highly persistent expectational AL term premium much in line with those obtained from no-arbitrage affine models (ACM and KW). Table 2 also shows the correlations of alternative measures of term premiums with the cyclical component of GDP (HP-GDP) computed with the HP filter (Hodrick and Prescott, 1997). Clearly, all term premiums are (mildly) countercyclical, in line with much of the relevant theoretical and empirical literature (e.g. Campbell and Cochrane, 1999; Cochrane and Piazzesi, 2005) as emphasized by Bauer, Rudebusch, and Wu, 2014; among others).

Table 2. 10-year term premium second moments (1990:3-2017:3)

	Total AL (baseline)	Expect. AL (baseline)	Expect. AL (SPF inflation)	Exo. AL (baseline)	VA	ACM	KW
Std. dev.	1.20	1.26	1.32	1.70	1.18	1.01	0.90
1st-order autoc.	0.92	0.97	0.98	0.97	0.94	0.95	0.96
Cross-correlations (1990:3-2017:3)							
Expect. AL (baseline)	0.05	1					
Expect. AL (with SPF inflation)	–	0.99	1				
Exo. AL (baseline)	0.70	-0.71	–	1			
VA	0.52	-0.37	-0.42	0.64	1		
ACM	0.71	0.23	0.14	0.33	0.69	1	
KW	0.55	-0.18	-0.23	0.52	0.94	0.83	1
HP-GDP	-0.45	-0.17	-0.10	-0.20	-0.19	-0.51	-0.26
Cross-correlations (2001:1-2017:3)							
Expect. AL (baseline)	0.32	1					
Expect. AL (with SPF inflation)	–	0.98	1				
Exo. AL (baseline)	0.59	-0.58	–	1			
VA	0.17	0.38	0.40	-0.18	1		
ACM	0.49	0.82	0.80	-0.28	0.52	1	
KW	0.20	0.71	0.74	-0.43	0.85	0.80	1
HP-GDP	-0.48	-0.30	-0.23	-0.15	-0.06	-0.56	-0.29

Notes: We do not show the cross-correlations of the expectational term premium estimated including the SPF average inflation in the set of observables with the estimates of the total and the exogenous term premium measures obtained in the baseline estimation that excludes SPF average inflation from the set of observables.

To bring to light differences and similarities across term premiums, Table 2 also shows the correlations between the six term premiums for the period 1990:3-2017:3 and for the most recent subsample 2001:1-2017:3. For the whole period (1990:3-2017:3), a low correlation (0.23) can be observed between the baseline expectational AL term premium and the ACM term premium. Meanwhile, the exogenous AL term premium captured by the estimated measurement error associated with the 10-year yield shows a moderate correlation (0.52) with the KW term premium, lower than that (0.83) of the two term premiums obtained from affine models (ACM and KW). Moreover, the exogenous measure of the 10-year AL term premium is more closely correlated with the other three measures of the term premium (VA, ACM, KW) than the expectational AL one, suggesting that model uncertainty brought about by the latter might not seem important when the whole sample period is considered. These correlations are in clear contrast with those obtained for the period 2001:1-2017:3, which includes the 2001 recession and the Great Recession. In this recent 17-year period, the correlations of the expectational term premium with the two affine term premiums are much higher. This is particularly true for the ACM term premium, with a high correlation at 0.82, while the correlation between the expectational AL and KW is lower at 0.71. In contrast, the exogenous measure of the 10-year AL term premium shows negative correlations with the ACM and KW measures (as well as with the VA and the expectational AL ones), indicating that it has performed poorly in recent times.[33] These results suggest that the misalignment of expectations between inflation and the policy interest rate captured by the expectational AL term premium with discretional beliefs has been especially substantial lately and that the ACM is able to capture such misaligned expectations somewhat better than the KW model. These findings challenge the view that term premium measures estimated from reduced-form (affine) models are *only* driven by risk as assumed in RE frameworks. In particular, misalignments in policy rate expectations due to a sort of bounded rationality

[33] As discussed above, this exogenous measurement error might be capable of capturing the component of the term premium defined as the additional return for the inflation and consumption risks associated with a long-term bond. Recall that this risk term premium component is totally ignored when the expectational term premium is characterized by considering the (linearized) consumption-based asset pricing equations (8).

(and uncertainty) also seem to be an important driving force behind the dynamics of term premium measures obtained from affine models in recent times.

This insight is explored further in Figure 5, which shows the dynamic correlation between inflation and the 10-year yield (black line) together with the dynamic means of (i) the 10-year expectational AL term premium (grey line); (ii) the estimated 10-year expected inflation path implied by the EHTS (red line),[34] computed as $\left[\frac{1}{40}\left(\sum_{k=0}^{39} E_t^{AL} r_{t+k}^{\{1\}}\right) - \frac{\left[E_t^{AL} x_{t+40} - x_t\right]}{40\left(1-\hat{\bar{h}}\right)}\right]$, where $\hat{\bar{h}}$ is the implied value of $\bar{h} = \frac{h}{\gamma}$ defined above, which is obtained from the estimated posterior means of h and γ; and (iii) the estimated 10-year expected inflation path (blue line) computed as $\frac{1}{40}\left(\sum_{k=1}^{40} E_t^{AL} \pi_{t+k}\right)$. These three time series are estimated as by-products of the baseline AL specification suggested here. These dynamic moments are computed using a 20-year rolling window. The horizontal axis indicates the first quarter of the corresponding rolling window. Since the early 1990s all windows have shown low dynamic correlation between inflation and the 10-year yield, evidencing that the Gibson paradox shown in recent literature for short-term interest rates (Casares and Vázquez, 2018) is carried over from the short end to the long end of the yield curve. As discussed above, the Gibson paradox seems to result in a disconnection between inflation and the policy rate and that disconnection is extended to some degree to the whole yield curve. The disconnection is likely to further result in a decoupling between the 10-year inflation expectation path and the 10-year path of inflation expectations embedded in the expectations of the future policy rate as shown by their divergent paths in Figure 5. These divergent paths have resulted in a sizable estimated 10-year expectational AL term premium in recent times. Notice also that these divergent paths of inflation expectations are rather similar to those shown in Figure 1, which were obtained using SPF (short-term) forecasts.

The question of how monetary policy featuring forward guidance may affect the expectational term premium may be raised. Forward guidance may certainly reduce the decoupling between inflation expectations of agents and the inflation expectations embedded in their

[34] That is, the 10-year expected inflation embedded at the policy rate expectations over the next 10 years.

policy rate expectations, but the evidence from the expectations reported in the SPF (as shown in Figure 1) and those implied by the AL-DSGE with discretional beliefs (Figure 5) casts doubt on the success of forward guidance on this front, especially during recession episodes surrounded by economic uncertainty signaled by expectational premium spikes.

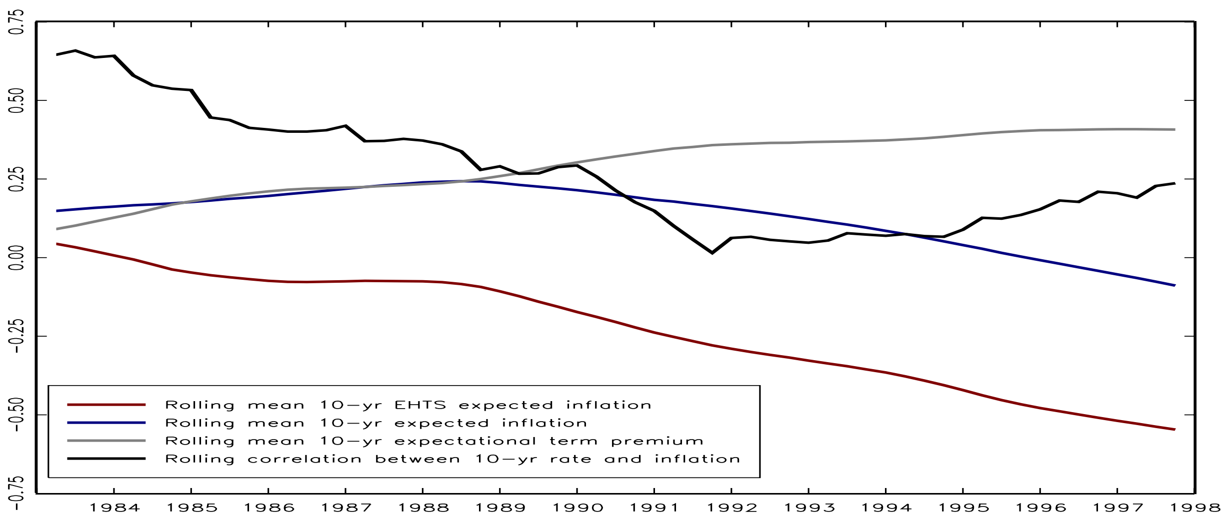

Figure 5. Expectational term premium and the Gibson paradox

Notes: The dynamic means and the correlation between inflation and the 10-year yield are computed using a 20-year rolling window. The dynamic mean of the expectational AL 10-year term premium is computed as $\frac{1}{40}\left(\sum_{k=1}^{40} E_t^{AL}\pi_{t+k}\right) - \left[\frac{1}{40}\left(\sum_{k=0}^{39} E_t^{AL} r_{t+k}^{\{1\}}\right) - \frac{[E_t^{AL}x_{t+40}-x_t]}{40\left(1-\hat{h}\right)}\right]$. The dynamic mean of the AL 10-year expected inflation is computed as $\frac{1}{40}\left(\sum_{k=1}^{40} E_t^{AL}\pi_{t+k}\right)$. The dynamic mean of the 10-year EHTS expected inflation is computed as $\left[\frac{1}{40}\left(\sum_{k=0}^{39} E_t^{AL} r_{t+k}^{\{1\}}\right) - \frac{[E_t^{AL}x_{t+40}-x_t]}{40\left(1-\hat{h}\right)}\right]$. Variables are measured in *quarterized* units.

Term premium variance decomposition

Table 3 shows the variance decomposition of relevant observable variables used in the estimation procedure and the expectational AL term premium. Each cell in this table reports two figures: First, the contribution to the forecast error variance for the 1-year forecast horizon of the corresponding variable; and second, the unconditional variance decomposition. The low level of persistence of the variables measured in growth rates implies that the variance decompositions at the short- and long-term horizons are fairly similar. However, the differences in variance decomposition across forecast horizons are much greater for the remaining variables, which are much more persistent, as shown in Figure 2 above. Focusing on the expectational AL term premium, it can be seen that the main drivers of its short-term fluctuations are monetary policy shocks (67.3%) and wage markup shocks (22.7%), while the contributions of other shocks are much smaller (e.g. the contribution the investment shock is 3.7%). The relative contributions of shocks change quite dramatically when the unconditional variance decomposition is analyzed. Thus, investment technology shocks explain 85.1% of the long-term fluctuations in the expectational AL term premium while the contributions of monetary and wage markup shocks drop substantially to 6.8% and 4.5%, respectively.

Table 3. Variance decomposition

	Δy	Δc	Δinv	Δw	l	π	r	$r^{AL\{4\}}$	$r^{AL\{40\}}$	$tp^{AL\{40\}}$
Shocks										
Productivity	10.9/10.5	0.6/0.6	0.0/0.0	0.3/0.3	29.3/12.1	0.9/0.2	1.2/3.2	1.6/3.49	0.1/0.1	2.1/0.5
Risk premium	24.8/26.9	83.6/82.0	0.8/1.0	3.8/4.0	27.5/8.8	13.4/10.4	0.1/4.8	0.3/3.8	2.0/1.5	1.9/0.2
Exog. spending	52.1/50.3	0.6/0.5	0.0/0.0	0.3/0.3	31.0/56.5	0.1/0.1	1.3/21.2	1.8/21.8	0.1/0.6	2.0/2.9
Investment	10.5/10.2	2.0/4.0	99.1/98.9	9.5/9.6	5.4/16.0	0.6/39.1	0.8/2.3	2.1/10.0	0.4/70.2	3.7/85.1
Monet. policy	0.0/0.0	0.1/0.2	0.0/0.0	1.1/1.1	0.1/0.1	3.1/1.8	96.2/67.8	75.3/54.7	15.1/8.5	67.3/6.8
Price mark-up	0.0/0.0	0.1/0.1	0.0/0.0	1.2/1.2	0.0/0.0	79.3/46.4	0.0/0.0	0.0/0.0	0.0/0.0	0.3/0.0
Wage mark-up	1.8/2.0	13.1/12.5	0.1/0.2	83.9/83.5	6.7/6.5	2.9/2.1	0.4/0.7	4.9/4.1	0.2/0.1	22.7/4.5
1-yr yield	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	14.0/2.3	0.0/0.0	0.0/0.0
10-yr yield	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	0.0/0.0	82.2/19.0	0.0/0.0

Notes: Each cell reports the contributions to the forecast error variance of the corresponding variable for the 1-year forecast horizon and the unconditional variance decomposition, respectively.

7 Conclusions

This paper provides an alternative explanation of the bond term premium based on a DSGE model featuring adaptive learning (AL) with discretional beliefs. In this AL framework, agents do not know the structure of the economy, so they face an important source of uncertainty: They do not know what the true model is or what the alternative sources of fundamental uncertainty are, and they have to learn how the economy behaves from the time series that they observe.

Following previous work by Aguilar and Vázquez (2021) and Vázquez and Aguilar (2021), this paper extends the AL model of Slobodyan and Wouters (2012a) by introducing the term structure of interest rates. But, in contrast to those papers, an endogenous expectational term premium is obtained under AL where discretional beliefs result in departures from the expectations hypothesis of the term structure. The term premium is usually viewed as a compensation for risk associated with investment in long-term bonds, but the expectational term premium suggested here is the result of model uncertainty, resulting in a misalignment between the inflation expectations held by agents and those which are embedded in short-term interest (policy rate) expectations. The estimated expectational AL term premium shares significant features with those estimated recently from no-arbitrage affine term structure models (e.g. Adrian, Crump and Moench, 2013) when misaligned expectations are sizable, and hence casts doubt on the effectiveness of forward guidance in monetary policy during recent recessions.

More generally, this paper contributes to an important research agenda: Including term structure in the DSGE framework. Ultimately, there is a need to find a macro-finance model that captures macro data, the term premium, expectations, and also uncertainty. Having such a workhorse model is especially important during quantitative easing, when monetary policy is an active market participant in markets with long-term maturities and risky assets.

References

- Adam, Klaus. 2005. "Learning to forecast and cyclical behavior of output and inflation." *Macroeconomic Dynamics* 9, 1-27.

- Adam, Klaus, and Albert Marcet. 2011. "Internal rationality, imperfect market knowledge and asset prices." *Journal of Economic Theory* 146, 1224-1252.

- Adam, Klaus, Albert Marcet, and Johannes Beutel. 2017. "Stock price booms and expected capital gains." *American Economic Review* 107, 2352-2408.

- Adam, Klaus, Albert Marcet, and Juan P. Nicolini. 2016. "Stock market volatility and learning." *Journal of Finance* 71, 33-82.

- Adrian, Tobias, Richard K. Crump, and Emanuel Moench. 2013. "Pricing the term structure with linear regressions." *Journal of Financial Economics* 110, 110-138.

- Aguilar, Pablo A., and Jesús Vázquez. 2021. "An estimated DSGE model with learning based on term structure information." *Macroeconomic Dynamics* 25, 1635–1665.

- Aguirre, Idoia, and Jesús Vázquez. 2020. "Learning, parameter variability, and swings in US macroeconomic dynamics." *Journal of Macroeconomics* 66, December, article 103240.

- Amisano, Gianni, and Oreste Tristani. 2023. "Monetary policy and long-term interest rates." *Quantitative Economics* 14, 689-716.

- Andreasen, Martin M., Jesús Fernández-Villaverde, and Juan F. Rubio-Ramírez. 2018. "The pruned state-space system for non-linear DSGE models: Theory and empirical applications." *Review of Economic Studies* 85, 1-49.

- Bansal, Ravi, and Amir Yaron. 2004. "Risks for the long run: A potential resolution of asset pricing puzzles." *Journal of Finance* 59, 1481–1509.

- Barillas, Francisco, Lars P. Hansen, and Thomas J. Sargent. 2009. "Doubts or variability?" *Journal of Economic Theory* 144, 2388-2418.

- Barillas, Francisco, and Kristoffer Nimark. 2019. "Speculation and the bond market: An empirical no-arbitrage framework." *Management Science* 65, 4179-4203.

- Bauer, Michael D., Glenn D. Rudebusch, and Jing C. Wu. 2014. "Term premia and inflation uncertainty: Empirical evidence from an international panel dataset: Comment." *American Economic Review* 104, 323-333.

- Bhansali, Rajendra J. 2002. "Multi-step forecasting," in Michael P. Clements and David F. Hendry, eds., *A Companion to Economic Forecasting*. Oxford: Blackwell Publishing, pp. 206-221.

- Bloom, Nicholas. 2014. "Fluctuations in uncertainty." *Journal of Economic Perspectives* 28, 153-176.

- Branch, William A., and George W. Evans. 2006. "A simple recursive forecasting model." *Economics Letters* 91, 158-66.

- Branch, William A., George W. Evans, and Bruce McGough. 2013. "Finite horizon learning," in Thomas J. Sargent and Jouko Vilmunen, eds., *Macroeconomics at the Service of Public Policy*, (Chapter 8) Oxford: Oxford University Press,.

- Calvo, Guillermo A. 1983. "Staggered pricing in a utility-maximizing framework." *Journal of Monetary Economics* 12, 383-396.

- Campbell, John Y., and John H. Cochrane. 1999. "By force of habit: A consumption-based explanation of aggregate stock market behavior." *Journal of Political Economy* 107, 205-251

- Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian. 2017. "Targeting long rates in a model with segmented markets." *American Economic Journal: Macroeconomics* 9, 205–242.

- Casares, Miguel, and Jesús Vázquez. 2018. "The swings of U.S. inflation and the Gibson paradox." *Economic Inquiry* 56, 799–820.

- Cochrane, John H., and Monika Piazzesi. 2005. “Bond risk premia.” *American Economic Review* 95, 138-160.

- Cohen, Benjamin H., Peter Hördahl, and Dora Xia. 2018. “Term premia: models and some stylised facts.” *BIS Quarterly Review,* September.

- Cogley, Timothy, Thomas J. Sargent, and Paolo. Surico. 2012. “The return of the Gibson paradox.” Manuscript, New York University.

- De Graeve, Ferre, Marina Emiris and Rafael Wouters. 2009. “A structural decomposition of the US yield curve.” *Journal of Monetary Economics* 56, 545-559.

- Del Negro, Marco, Domenico Giannone, Marc Giannoni, and Andrea Tambalotti. 2017. “Safety, liquidity, and the natural rate of interest.” *Brooking Papers on Economic Activity* Conference drafts.

- Dew-Becker, Ian, 2014. “Bond pricing with a time-varying price of risk in an estimated medium-scale Bayesian DSGE model.” *Journal of Money, Credit and Banking* 46, 837-888.

- Epstein, Larry G., and Stanley E. Zin. 1989. “Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework.” *Econometrica* 57, 937-969.

- Estrella, Arturo, and Gikas A. Hardouvelis. 1991. “The term structure as a predictor of real economic activity.” *Journal of Finance* 46, 555-576.

- Estrella, Arturo, and Frederic S. Mishkin. 1997. “The predictive power of the term structure of interest rates in Europe and the United States: Implications for the European Central Bank.” *European Economic Review* 41, 1375-1401.

- Eusepi, Stefano, and Bruce Preston. 2011. “Expectations, learning, and business cycle fluctuations”. *American Economic Review* 101, 2844-2872.

- Evans, George W., and Seppo Honkapohja. 2012. *Learning and Expectations in Macroeconomics*. Princeton University Press.

- Froot, Kenneth A., and Takatoshi Ito. 1989. "On the consistency of short-run and long-run exchange rate expectations". *Journal of International Money and Finance* 8, 487-510.
- Gertler Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics* 58, 17-34.
- Greenwood, Robin, Samuel G. Hanson, and Jeremy C. Stein. 2015. "A comparative-advantage approach to government debt maturity." *Journal of Finance* 70, 1683-1722.
- Gürkaynak, Refet S., and Jonathan H. Wright. 2012. "Macroeconomics and the term structure." *Journal of Economic Literature* 50, 331-367.
- Hodrick, Robert J., and Edward C. Prescott. 1997. "Postwar U.S. business cycles: An empirical investigation." *Journal of Money, Credit, and Banking* 29, 1-16.
- Hommes, Cars, and Gerhard Sorger. 1998. Consistent expectations equilibria. *Macroeconomic Dynamics* 2, 287-321.
- Honkapohja, Seppo, Kaushik Mitra, and George W. Evans. 2012. "Notes on agents' behavioral rules under adaptive learning and studies of monetary policy". CDMA11/02 working paper, University of St Andrews.
- Hördahl, Peter, and Oreste Tristani. 2014. Inflation risk premia in the Euro Area and the United States. *International Journal of Central Banking* 10, 1-47.
- Jordà, Oscar. 2005. "Estimation and inference of impulse responses by local projections." *American Economic Review* 95, 161-182.
- Kim, Don H., and Athanasios Orphanides. 2012. "Term structure estimation with survey data on interest rate forecasts." *Journal of Financial and Quantitative Analysis* 47, 241-272.
- Kim, Don H., and Jonathan H. Wright. 2005. "An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates." Board of Governors of the Federal Reserve System Finance and Economics Discussion Paper 2005-33.

- Kliem, Martin, and Alexander Meyer-Gohde. 2022. "(Un)expected monetary policy shocks and term premia." *Journal of Applied Econometrics* 37, 477-499.

- Knight, Frank H. 1921. *Risk, Uncertainty, and Profit*. Boston, MA: Hart, Schaffner & Marx; Houghton Miffl in Company.

- Kozicki, Sharon, and Peter A. Tinsley. 2005. "What do you expect? Imperfect policy credibility and tests of the expectations hypothesis." *Journal of Monetary Economics* 52, 421-447.

- Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012. "The aggregate demand for Treasury debt." *Journal of Political Economy* 120, 233-267.

- Li, Canlin, Andrew Meldrum, and Marius Rodriguez. 2017. "Robustness of long-maturity term premium estimates." *FEDS Notes. Washington: Board of Governors of the Federal Reserve System*, April 3.

- Lucas, Robert E. Jr. 2003. "Macroeconomic priorities." *American Economic Review, Papers and Proceedings* 93, 1-14.

- McCallum, Bennett T. 1994. "Monetary policy and the term structure of interest rates". NBER Working Paper 4938. Reprinted in the *Federal Reserve Bank of Richmond Economic Quarterly*, Volume 91/4 Fall 2005.

- Milani, Fabio. 2007. "Expectations, learning and macroeconomic persistence." *Journal of Monetary Economics* 54, 2065-2082.

- Milani, Fabio. 2008. "Learning, monetary policy rules, and macroeconomic stability." *Journal of Economic Dynamics and Control* 32, 3148-3165.

- Milani, Fabio. 2011. "Expectation shocks and learning as drivers of the business cycle." *Economic Journal* 121, 379-401.

- Mishkin, Frederic S. 1990. "The information in the longer maturity term structure about future inflation." *Quarterly Journal of Economics* 105, 815-828.

- Mouabbi, Sarah, and Jean-Guillaume Sahuc. 2019. "Evaluating the macroeconomic effects of the ECB's unconventional monetary policies." *Journal of Money, Credit and Banking* 51, 831-858.

- Orphanides, Athanasios, and John C. Williams. 2005. "Inflation scares and forecast-based monetary policy." *Review of Economic Dynamics* 8, 498-527.

- Preston, Bruce. 2005. "Learning about monetary policy rules when long-horizon expectations matter." *International Journal of Central Banking* 1, 81-126.

- Rudebusch, Glenn D., and Eric T. Swanson. 2008. "Examining the bond premium puzzle with a DSGE model". *Journal of Monetary Economics* 55, S111–S126.

- Rudebusch, Glenn D., and Eric T. Swanson. 2012. "The bond premium in a DSGE model with long-run real and nominal risks". *American Economic Journal: Macroeconomics* 4, 105–143.

- Rychalovska, Yuliya, Sergey Slobodyan, and Rafael Wouters. 2016. "Financial frictions, expectations and business cycle: evidence from an estimated DSGE model. National Bank of Belgium, *mimeo*.

- Sargent, Thomas. J. 1991. Equilibrium with signal extraction from endogenous variables. *Journal of Economic Dynamics and Control* 15, 245-273.

- Sims, Eric, and Jing C. Wu. 2021. "Evaluating central banks' tool kit: Past, present, and future". *Journal of Monetary Economics* 118, 135–160.

- Sinha, Arunima. 2015. "Government debt, learning and the term structure." *Journal of Economic Dynamics and Control* 53, 268-289.

- Sinha, Arunima. 2016. "Learning and the yield curve." *Journal of Money, Credit, and Banking* 48, 513-547.

- Slobodyan, Sergey, and Rafael Wouters. 2008. "Estimating a medium–scale DSGE model with expectations based on small forecasting models." *Mimeo.*

- Slobodyan, Sergey, and Rafael Wouters. 2012a. "Learning in a medium-Scale DSGE model with expectations based on small forecasting models." *American Economic Journal: Macroeconomics* 4, 65-101.

- Slobodyan, Sergey, and Rafael Wouters. 2012b. "Learning in an estimated medium-Scale DSGE model." *Journal of Economic Dynamics and Control* 36, 22-46.

- Smets, Frank R., and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American Economic Review* 97, 586-606.

- Stark, Tom. 2013. "SPF panelists' forecasting methods: A note on the aggregate results of a November 2009 special survey." Federal Reserve Bank of Philadelphia, Research Department Report.

- Van Binsbergen, Jules H., Jesús Fernández-Villaverde, Ralph S. J. Koijen, and Juan Rubio-Ramírez. 2012. "The term structure of interest rates in a DSGE model with recursive preferences". *Journal of Monetary Economics* 59, 634-648.

- Vázquez, Jesús, and Aguilar, Pablo. 2021. "Adaptive learning with term structure information." *European Economic Review* 134, May, article 103689.

- Vázquez, Jesús, Ramón María-Dolores, and Juan M. Londoño. 2013. "On the informational role of term structure in the US monetary policy rule." *Journal of Economic Dynamics and Control* 37, 1852-1871.

- Walsh, Carl E. 2010. *Monetary Theory and Practice*. The MIT Press: Cambridge, Massachusetts.

- Wu, Jing. C., and Fan D. Xia. 2016. Measuring the macroeconomic impact of monetary policy at the zero lower bound." *Journal of Money, Credit, and Banking* 48, 253-291.

- Wu, Jing. C., and Ji Zhang. 2019. "A shadow rate new Keynesian model." *Journal of Economic Dynamics and Control* 107, October, article 103728.

Supplementary appendix (Not intended for publication)

Part 1

The estimated DSGE model under AL is given by

- using the definition of the expectational term premium (9), the optimality condition (8) for $j = 1, 4, 40$ can be written as:

$$x_t = E_t^{AL} x_{t+j} - \left(1 - \bar{h}\right) \left[j r_t^{AL-EH\{j\}} - \sum_{k=1}^{j} E_t^{AL} \pi_{t+k} + j \left(tp_t^{\{j\}} \right) \right],$$

where $r_t^{AL-EH\{j\}} = \frac{1}{j} \sum_{k=0}^{j-1} E_t^{AL} r_{t+k}^{\{1\}}$, E_t^{AL} denotes the AL operator under discretional beliefs, and $tp_t^{\{1\}} = \varepsilon_t^b$ is a risk premium structural shock, as in equation (2) of Smets and Wouters (2007), representing a wedge between the interest rate controlled by the central bank and the return on 1-period assets held by the households.

- the policy rule (17)

$$r_t^{\{1\}} = \rho_r r_{t-1}^{\{1\}} + (1 - \rho_r) \left[r_\pi E_t^{AL} \pi_{t+1} + r_y \hat{y}_t \right] + r_{\Delta y} \Delta \hat{y}_t + r_{sp} sp_t^{\{4\}} + \varepsilon_t^r,$$

- and the set of the remaining log-linearized dynamic equations in Slobodyan and Wouters (2012a):
  - Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{19}$$

    where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta) \frac{K}{Y}$, and $z_y = r^k \frac{K}{Y}$ are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.
  - Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t^{AL} i_{t+1} + i_2 q_t + \varepsilon_t^i, \tag{20}$$

where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{(1+\overline{\beta})\gamma^2\varphi}$ with $\overline{\beta} = \beta\gamma^{(1-\sigma_c)}$.

– Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t^{AL} q_{t+1} + (1-q_1) E_t^{AL} r_{t+1}^k - \left(r_t^{\{1\}} - E_t^{AL}\pi_{t+1}\right) + c_3^{-1}\varepsilon_t^b, \tag{21}$$

where $q_1 = \overline{\beta}\gamma^{-1}(1-\delta) = \frac{(1-\delta)}{\left(r^k+1-\delta\right)}$.

– Log-linearized aggregate production function:

$$y_t = \Phi\left(\alpha k_t^s + (1-\alpha)l_t + \varepsilon_t^a\right), \tag{22}$$

where $\Phi = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and α is the capital-share in the production function.[35]

– Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \tag{23}$$

– Capital utilization:

$$z_t = z_1 r_t^k, \tag{24}$$

where $z_1 = \frac{1-\psi}{\psi}$.

– Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1-k_1)i_t + k_2\varepsilon_t^i, \tag{25}$$

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)\left(1+\overline{\beta}\right)\gamma^2\varphi$.

– Marginal cost:

$$mc_t = (1-\alpha)w_t + \alpha r_t^k - \varepsilon_t^a. \tag{26}$$

[35]From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

– New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t^{AL} \pi_{t+1} - \pi_3 mc_t + \pi_4 \varepsilon_t^p, \tag{27}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, $\pi_3 = \frac{A}{1+\overline{\beta}\iota_p}\left[\frac{(1-\overline{\beta}\xi_p)(1-\xi_p)}{\xi_p}\right]$, and $\pi_4 = \frac{1+\overline{\beta}\iota_p}{1+\overline{\beta}\iota_p}$. The coefficient of the curvature of the Kimball goods market aggregator, included in the definition of A, is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).

– Optimal demand for capital by firms:

$$-\left(k_t^s - l_t\right) + w_t = r_t^k. \tag{28}$$

– Wage markup equation:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1-h/\gamma}\left(c_t - (h/\gamma)\, c_{t-1}\right)\right). \tag{29}$$

– Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1)\left(E_t^{AL} w_{t+1} + E_t^{AL}\pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w. \tag{30}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{(1-\overline{\beta}\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$ and a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007)

Part 2

This part of the supplementary appendix analyzes the performance of the baseline- and the VA-AL specifications in reproducing selected second-moment statistics obtained from actual data as shown in Table A.1. More precisely, the simulated data is generated by carrying out a stochastic simulation of the model using the mean of the posterior distribution of each estimated parameter. I carry out 5,000 stochastic simulations of equal size of the sample period in order to compute the mean and the 90% simulated credible set of each simulated statistic. I focus on five types of moment: standard deviations, first-order autocorrelations, and the correlations of the alternative variables with output growth, inflation, and the short-term (policy) interest rate, respectively.

For standard deviations, the two estimated AL models are able to match the volatility of all variables reasonably well, at least qualitatively. For first-order autocorrelation, the two AL specifications reproduce the low/moderate persistence of the growth rates of most real variables rather well. It also does a good job in reproducing the high persistence of hours worked and the nominal variables. For correlations with output growth, the AL models capture the moderate/high correlation between consumption and output growth rates and the low correlation of output growth with hours worked and to some extent with inflation. For the rest of the variables, the estimated correlation with output growth is in general lower than the actual correlation. The model does also a good job in characterizing the correlations of the alternative variables with both inflation and the short-term interest rate. These results are rather similar for the two AL specifications.

Table A.1. Actual and simulated second moments (1983:2-2017:3)

	Δy	Δc	Δinv	Δw	l	π	r	$r^{\{4\}}$	$r^{EH\{4\}}$	$r^{\{40\}}$
Actual data										
Std. dev.	0.62*	0.60*	1.88^{+}	0.86^{+}	4.08^{+}	0.25	0.87^{*+}	0.76^{*+}	0.77^{+}	0.67*
Autocor.	0.41^{+}	0.37^{*+}	0.67	-0.19	0.99	0.61	0.99^{*+}	0.99	0.99	0.99
Cor.(Δy)	1.0	0.70^{*+}	0.69	-0.02*	0.15^{*+}	0.14^{+}	0.28^{+}	0.35	0.37	0.37
Cor.(π)	0.14^{+}	0.09^{*+}	0.11^{*+}	-0.14	0.29^{*+}	1.0	0.54^{*+}	0.56^{*+}	0.56^{*+}	0.54^{*+}
Cor.(r)	0.28^{+}	0.30^{+}	0.04^{*+}	0.11^{*+}	0.71	0.54^{*+}	1.0	0.98*	0.97	0.90
Simulated data										
Std. dev.	0.64/0.78	0.65/0.77	1.44/1.77	1.11/0.88	1.88/2.81	0.75/0.56	0.63/0.67	0.61/0.60	0.39/0.57	0.44/0.43
	(0.57,0.70)	(0.58,0.72)	(1.27,1.63)	(1.00,1.23)	(1.21,2.78)	(0.48,1.14)	(0.39,0.95)	(0.38,0.90)	(0.25,0.58)	(0.26,0.69)
Autocor.	0.14/0.33	0.23/0.53	0.46/0.55	0.19/0.02	0.94/0.97	0.93/0.89	0.98/0.97	0.96/0.94	0.95/0.94	0.95/0.95
	(0.00,0.28)	(0.08,0.38)	(0.34,0.58)	(0.05,0.33)	(0.88,0.98)	(0.85,0.97)	(0.95,0.99)	(0.92,0.98)	(0.90,0.98)	(0.89,0.98)
Cor.(Δy)	1.0	0.69/0.78	0.28/0.27	0.03/0.15	0.11/0.09	-0.03/0.02	-0.04/0.12	-0.05/-0.02	-0.01/0.01	-0.02/0.06
	—	(0.61,0.76)	(0.14,0.41)	(-0.11,0.17)	(-0.01,0.23)	(-0.16,0.10)	(-0.18,0.10)	(-0.19,0.09)	(-0.15,0.13)	(-0.15,0.11)
Cor.(π)	-0.03/0.02	0.00/0.02	-0.09/0.10	-0.05/0.05	0.18/0.06	1.0	0.21/0.71	0.09/0.75	0.37/0.69	0.32/0.45
	(-0.16,0.10)	(-0.12,0.12)	(-0.29,0.11)	(-0.21,0.12)	(-0.43,0.68)	—	(-0.39,0.71)	(-0.49,0.63)	(-0.15,0.78)	(-0.29,0.77)
Cor.(r)	-0.04/0.12	-0.04/0.17	-0.01/-0.04	0.01/0.03	0.18/0.07	0.21/0.71	1.0	0.94/0.80	0.84/0.68	0.42/0.44
	(-0.18,0.10)	(-0.18,0.11)	(-0.22,0.21)	(-0.15,0.18)	(-0.39,0.67)	(-0.39,0.71)	—	(0.86,0.98)	(0.62,0.96)	(-0.18,0.83)

Note: An asterisk (a cross) in the upper panel indicates that the actual second moment statistic lies within the estimated credible set for the baseline (VA)-AL model. Each cell in the bottom panel shows the posterior estimates for the baseline- and the VA-AL specifications, respectively. The bottom panel only shows the credible sets for the baseline specification in parentheses. The credible sets for the VA-AL specification are available upon request.

Part 3

Table A.2.1: Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
				Baseline			VA-AL		
Log-likelihood				-371.03			-445.73		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
φ: cost of adjusting capital	Normal	4.00	1.50	2.61	2.33	2.79	2.65	2.62	2.67
h: habit formation	Beta	0.70	0.10	0.26	0.23	0.29	0.71	0.69	0.72
σ_l : Frisch elasticity	Normal	2.00	0.50	2.49	2.40	2.62	1.86	1.85	1.87
ξ_p: price Calvo probability	Beta	0.50	0.10	0.66	0.61	0.71	0.63	0.60	0.65
ξ_w: wage Calvo probability	Beta	0.50	0.10	0.24	0.21	0.27	0.59	0.57	0.62
ι_w: wage indexation	Beta	0.50	0.15	0.52	0.45	0.58	0.48	0.36	0.60
ι_p: price indexation	Beta	0.50	0.15	0.17	0.11	0.23	0.14	0.10	0.18
ψ: capital utilization adjusting cost	Beta	0.50	0.15	0.75	0.69	0.81	0.03	0.02	0.03
Φ : steady state price mark-up	Normal	1.25	0.12	1.27	1.18	1.35	1.37	1.29	1.45
r_π: policy rule inflation	Normal	1.50	0.25	1.56	1.45	1.64	1.36	1.30	1.43
ρ_r: policy rule smoothing	Beta	0.75	0.10	0.97	0.96	0.98	0.86	0.85	0.88
r_y: policy rule output gap	Beta	0.12	0.05	0.16	0.13	0.20	0.005	0.002	0.007
$r_{\Delta y}$: policy rule output gap growth	Beta	0.12	0.05	0.03	0.02	0.03	0.008	0.006	0.010
r_{sp}: policy rule term spread	Normal	0.12	0.05	0.18	0.15	0.21	0.17	0.15	0.19
π: steady-state inflation	Gamma	0.62	0.10	0.63	0.56	0.71	0.61	0.58	0.64
$100(\beta^{-1}-1)$: steady-state rate of disc.	Gamma	0.25	0.10	0.32	0.28	0.36	0.26	0.21	0.31
$\overline{r}^{SPF}$: mean of SPF rate.	Uniform	1.00	2.00	1.10	0.98	1.24	0.94	0.77	1.11
l: steady-state labor	Normal	0.00	2.00	-0.62	-1.25	-0.12	-0.57	-0.77	-0.26
γ: one plus steady-state growth rate	Normal	0.40	0.10	0.43	0.41	0.45	0.42	0.41	0.43
α: capital share	Normal	0.30	0.05	0.12	0.10	0.14	0.11	0.10	0.13
μ_π: expectations inflation coef.	Beta	0.90	0.15	0.71	0.66	0.78	—	—	—
μ_c: expectations consumption coef.	Beta	0.90	0.15	0.99	0.95	1.0	—	—	—
μ_r: expectations policy rate coef.	Beta	0.90	0.15	0.97	0.92	1.0	—	—	—
ρ: learning parameter	Beta	0.50	0.29	0.93	0.92	0.94	0.949	0.943	0.954

Table A.2.2: Priors and estimated posteriors of the structural shock process parameters

	Priors			Posteriors					
				Baseline			VA-AL		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a: Std. dev. productivity innov.	Invgamma	0.10	2.00	0.44	0.40	0.48	0.45	0.42	0.48
σ_b: Std. dev. risk premium innov.	Invgamma	0.10	2.00	0.60	0.54	0.67	0.26	0.25	0.28
σ_g: Std. dev. exogenous spending innov.	Invgamma	0.10	2.00	0.39	0.36	0.42	0.38	0.34	0.41
σ_i: Std. dev. investment innov.	Invgamma	0.10	2.00	1.36	1.29	1.43	1.11	1.08	1.15
σ_R: Std. dev. monetary policy innov.	Invgamma	0.10	2.00	0.11	0.09	0.12	0.106	0.097	0.115
σ_p: Std. dev. price mark-up innov.	Invgamma	0.10	2.00	0.23	0.21	0.25	0.19	0.18	0.21
σ_w: Std. dev. wage mark-up innov.	Invgamma	0.10	2.00	1.61	1.54	1.69	0.94	0.88	0.99
$\sigma_{\epsilon_r^{\{4\}}}$: Std. dev. 1-yr TP innov.	Uniform	2.00	2.00	0.35	0.32	0.39	0.11	0.10	0.12
$\sigma_{\epsilon_r^{\{40\}}}$: Std. dev. 10-yr TP innov.	Uniform	2.00	2.00	0.41	0.37	0.45	0.11	0.09	0.12
$\sigma_{\epsilon^{SPF}}$: Std. dev. SPF mesmt. error	Uniform	0.10	2.00	0.069	0.064	0.075	0.11	0.10	0.12
ρ_a: Autoreg. coef. product. shock	Beta	0.50	0.20	0.96	0.94	0.98	0.98	0.96	0.99
ρ_b: Autoreg. coef. risk shock	Beta	0.50	0.20	0.90	0.88	0.92	0.42	0.39	0.45
ρ_g: Autoreg. coef. exog. spen. shock	Beta	0.50	0.20	0.988	0.980	0.995	0.97	0.95	0.99
ρ_i: Autoreg. coef. invest. shock	Beta	0.50	0.20	0.97	0.96	0.98	0.95	0.94	0.96
ρ_R: Autoreg. coef. policy shock	Beta	0.50	0.20	0.52	0.49	0.58	0.74	0.71	0.77
ρ_p: Autoreg. coef. price mkup shock	Beta	0.50	0.20	0.98	0.96	0.99	0.95	0.94	0.97
ρ_w: Autoreg. coef. wage mkup shock	Beta	0.50	0.20	0.96	0.94	0.98	0.957	0.950	0.965
μ_p: MA coef. price markup shock	Beta	0.50	0.20	0.47	0.41	0.52	0.58	0.54	0.63
μ_w: MA coef. wage markup shock	Beta	0.50	0.20	0.17	0.11	0.21	0.49	0.43	0.54
$\rho_{\{4\}}$: Autoreg. coef. 1-yr TP	Beta	0.50	0.20	0.76	0.73	0.78	0.95	0.94	0.96
$\rho_{\{40\}}$: Autoreg. coef. 1-yr TP	Beta	0.50	0.20	0.94	0.91	0.96	0.97	0.96	0.99
ρ_{SPF}: Autoreg. coef. SPF mesmt. error	Beta	0.50	0.20	0.94	0.91	0.97	0.96	0.94	0.97
ρ_{ga}: interact. product. and spend. shocks	Beta	0.50	0.25	0.40	0.35	0.45	0.52	0.47	0.58

References

- Adam, Klaus. 2005. "Learning to forecast and cyclical behavior of output and inflation." *Macroeconomic Dynamics* 9, 1-27.

- Adam, Klaus, and Albert Marcet. 2011. "Internal rationality, imperfect market knowledge and asset prices." *Journal of Economic Theory* 146, 1224-1252.

- Adam, Klaus, Albert Marcet, and Johannes Beutel. 2017. "Stock price booms and expected capital gains." *American Economic Review* 107, 2352-2408.

- Adam, Klaus, Albert Marcet, and Juan P. Nicolini. 2016. "Stock market volatility and learning." *Journal of Finance* 71, 33-82.

- Adrian, Tobias, Richard K. Crump, and Emanuel Moench. 2013. "Pricing the term structure with linear regressions." *Journal of Financial Economics* 110, 110-138.

- Aguilar, Pablo A., and Jesús Vázquez. 2021. "An estimated DSGE model with learning based on term structure information." *Macroeconomic Dynamics* 25, 1635–1665.

- Aguirre, Idoia, and Jesús Vázquez. 2020. "Learning, parameter variability, and swings in US macroeconomic dynamics." *Journal of Macroeconomics* 66, December, article 103240.

- Amisano, Gianni, and Oreste Tristani. 2023. "Monetary policy and long-term interest rates." *Quantitative Economics* 14, 689-716.

- Andreasen, Martin M., Jesús Fernández-Villaverde, and Juan F. Rubio-Ramírez. 2018. "The pruned state-space system for non-linear DSGE models: Theory and empirical applications." *Review of Economic Studies* 85, 1-49.

- Bansal, Ravi, and Amir Yaron. 2004. "Risks for the long run: A potential resolution of asset pricing puzzles." *Journal of Finance* 59, 1481–1509.

- Barillas, Francisco, Lars P. Hansen, and Thomas J. Sargent. 2009. "Doubts or variability?" *Journal of Economic Theory* 144, 2388-2418.

- Barillas, Francisco, and Kristoffer Nimark. 2019. "Speculation and the bond market: An empirical no-arbitrage framework." *Management Science* 65, 4179-4203.

- Bauer, Michael D., Glenn D. Rudebusch, and Jing C. Wu. 2014. "Term premia and inflation uncertainty: Empirical evidence from an international panel dataset: Comment." *American Economic Review* 104, 323-333.

- Bhansali, Rajendra J. 2002. "Multi-step forecasting," in Michael P. Clements and David F. Hendry, eds., *A Companion to Economic Forecasting*. Oxford: Blackwell Publishing, pp. 206-221.

- Bloom, Nicholas. 2014. "Fluctuations in uncertainty." *Journal of Economic Perspectives* 28, 153-176.

- Branch, William A., and George W. Evans. 2006. "A simple recursive forecasting model." *Economics Letters* 91, 158-66.

- Branch, William A., George W. Evans, and Bruce McGough. 2013. "Finite horizon learning," in Thomas J. Sargent and Jouko Vilmunen, eds., *Macroeconomics at the Service of Public Policy*, (Chapter 8) Oxford: Oxford University Press,.

- Calvo, Guillermo A. 1983. "Staggered pricing in a utility-maximizing framework." *Journal of Monetary Economics* 12, 383-396.

- Campbell, John Y., and John H. Cochrane. 1999. "By force of habit: A consumption-based explanation of aggregate stock market behavior." *Journal of Political Economy* 107, 205-251

- Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian. 2017. "Targeting long rates in a model with segmented markets." *American Economic Journal: Macroeconomics* 9, 205–242.

- Casares, Miguel, and Jesús Vázquez. 2018. "The swings of U.S. inflation and the Gibson paradox." *Economic Inquiry* 56, 799–820.

- Cochrane, John H., and Monika Piazzesi. 2005. "Bond risk premia." *American Economic Review* 95, 138-160.

- Cohen, Benjamin H., Peter Hördahl, and Dora Xia. 2018. "Term premia: models and some stylised facts." *BIS Quarterly Review,* September.

- Cogley, Timothy, Thomas J. Sargent, and Paolo. Surico. 2012. "The return of the Gibson paradox." Manuscript, New York University.

- De Graeve, Ferre, Marina Emiris and Rafael Wouters. 2009. "A structural decomposition of the US yield curve." *Journal of Monetary Economics* 56, 545-559.

- Del Negro, Marco, Domenico Giannone, Marc Giannoni, and Andrea Tambalotti. 2017. "Safety, liquidity, and the natural rate of interest." *Brooking Papers on Economic Activity* Conference drafts.

- Dew-Becker, Ian, 2014. "Bond pricing with a time-varying price of risk in an estimated medium-scale Bayesian DSGE model." *Journal of Money, Credit and Banking* 46, 837-888.

- Epstein, Larry G., and Stanley E. Zin. 1989. "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework." *Econometrica* 57, 937-969.

- Estrella, Arturo, and Gikas A. Hardouvelis. 1991. "The term structure as a predictor of real economic activity." *Journal of Finance* 46, 555-576.

- Estrella, Arturo, and Frederic S. Mishkin. 1997. "The predictive power of the term structure of interest rates in Europe and the United States: Implications for the European Central Bank." *European Economic Review* 41, 1375-1401.

- Eusepi, Stefano, and Bruce Preston. 2011. "Expectations, learning, and business cycle fluctuations". *American Economic Review* 101, 2844-2872.

- Evans, George W., and Seppo Honkapohja. 2012. *Learning and Expectations in Macroeconomics*. Princeton University Press.

- Froot, Kenneth A., and Takatoshi Ito. 1989. "On the consistency of short-run and long-run exchange rate expectations". *Journal of International Money and Finance* 8, 487-510.

- Gertler Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics* 58, 17-34.

- Greenwood, Robin, Samuel G. Hanson, and Jeremy C. Stein. 2015. "A comparative-advantage approach to government debt maturity." *Journal of Finance* 70, 1683-1722.

- Gürkaynak, Refet S., and Jonathan H. Wright. 2012. "Macroeconomics and the term structure." *Journal of Economic Literature* 50, 331-367.

- Hodrick, Robert J., and Edward C. Prescott. 1997. "Postwar U.S. business cycles: An empirical investigation." *Journal of Money, Credit, and Banking* 29, 1-16.

- Hommes, Cars, and Gerhard Sorger. 1998. Consistent expectations equilibria. *Macroeconomic Dynamics* 2, 287-321.

- Honkapohja, Seppo, Kaushik Mitra, and George W. Evans. 2012. "Notes on agents' behavioral rules under adaptive learning and studies of monetary policy". CDMA11/02 working paper, University of St Andrews.

- Hördahl, Peter, and Oreste Tristani. 2014. Inflation risk premia in the Euro Area and the United States. *International Journal of Central Banking* 10, 1-47.

- Jordà, Oscar. 2005. "Estimation and inference of impulse responses by local projections." *American Economic Review* 95, 161-182.

- Kim, Don H., and Athanasios Orphanides. 2012. "Term structure estimation with survey data on interest rate forecasts." *Journal of Financial and Quantitative Analysis* 47, 241-272.

- Kim, Don H., and Jonathan H. Wright. 2005. "An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates." Board of Governors of the Federal Reserve System Finance and Economics Discussion Paper 2005-33.

- Kliem, Martin, and Alexander Meyer-Gohde. 2022. "(Un)expected monetary policy shocks and term premia." *Journal of Applied Econometrics* 37, 477-499.

- Knight, Frank H. 1921. *Risk, Uncertainty, and Profit*. Boston, MA: Hart, Schaffner & Marx; Houghton Miffl in Company.

- Kozicki, Sharon, and Peter A. Tinsley. 2005. "What do you expect? Imperfect policy credibility and tests of the expectations hypothesis." *Journal of Monetary Economics* 52, 421-447.

- Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2012. "The aggregate demand for Treasury debt." *Journal of Political Economy* 120, 233-267.

- Li, Canlin, Andrew Meldrum, and Marius Rodriguez. 2017. "Robustness of long-maturity term premium estimates." *FEDS Notes. Washington: Board of Governors of the Federal Reserve System*, April 3.

- Lucas, Robert E. Jr. 2003. "Macroeconomic priorities." *American Economic Review, Papers and Proceedings* 93, 1-14.

- McCallum, Bennett T. 1994. "Monetary policy and the term structure of interest rates". NBER Working Paper 4938. Reprinted in the *Federal Reserve Bank of Richmond Economic Quarterly*, Volume 91/4 Fall 2005.

- Milani, Fabio. 2007. "Expectations, learning and macroeconomic persistence." *Journal of Monetary Economics* 54, 2065-2082.

- Milani, Fabio. 2008. "Learning, monetary policy rules, and macroeconomic stability." *Journal of Economic Dynamics and Control* 32, 3148-3165.

- Milani, Fabio. 2011. "Expectation shocks and learning as drivers of the business cycle." *Economic Journal* 121, 379-401.

- Mishkin, Frederic S. 1990. "The information in the longer maturity term structure about future inflation." *Quarterly Journal of Economics* 105, 815-828.

- Mouabbi, Sarah, and Jean-Guillaume Sahuc. 2019. “Evaluating the macroeconomic effects of the ECB’s unconventional monetary policies.” *Journal of Money, Credit and Banking* 51, 831-858.

- Orphanides, Athanasios, and John C. Williams. 2005. “Inflation scares and forecast-based monetary policy.” *Review of Economic Dynamics* 8, 498-527.

- Preston, Bruce. 2005. “Learning about monetary policy rules when long-horizon expectations matter.” *International Journal of Central Banking* 1, 81-126.

- Rudebusch, Glenn D., and Eric T. Swanson. 2008. “Examining the bond premium puzzle with a DSGE model”. *Journal of Monetary Economics* 55, S111–S126.

- Rudebusch, Glenn D., and Eric T. Swanson. 2012. “The bond premium in a DSGE model with long-run real and nominal risks”. *American Economic Journal: Macroeconomics* 4, 105–143.

- Rychalovska, Yuliya, Sergey Slobodyan, and Rafael Wouters. 2016. “Financial frictions, expectations and business cycle: evidence from an estimated DSGE model. National Bank of Belgium, *mimeo*.

- Sargent, Thomas. J. 1991. Equilibrium with signal extraction from endogenous variables. *Journal of Economic Dynamics and Control* 15, 245-273.

- Sims, Eric, and Jing C. Wu. 2021. “Evaluating central banks’ tool kit: Past, present, and future”. *Journal of Monetary Economics* 118, 135–160.

- Sinha, Arunima. 2015. “Government debt, learning and the term structure.” *Journal of Economic Dynamics and Control* 53, 268-289.

- Sinha, Arunima. 2016. “Learning and the yield curve.” *Journal of Money, Credit, and Banking* 48, 513-547.

- Slobodyan, Sergey, and Rafael Wouters. 2008. “Estimating a medium–scale DSGE model with expectations based on small forecasting models.” *Mimeo.*

- Slobodyan, Sergey, and Rafael Wouters. 2012a. "Learning in a medium-Scale DSGE model with expectations based on small forecasting models." *American Economic Journal: Macroeconomics* 4, 65-101.

- Slobodyan, Sergey, and Rafael Wouters. 2012b. "Learning in an estimated medium-Scale DSGE model." *Journal of Economic Dynamics and Control* 36, 22-46.

- Smets, Frank R., and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American Economic Review* 97, 586-606.

- Stark, Tom. 2013. "SPF panelists' forecasting methods: A note on the aggregate results of a November 2009 special survey." Federal Reserve Bank of Philadelphia, Research Department Report.

- Van Binsbergen, Jules H., Jesús Fernández-Villaverde, Ralph S. J. Koijen, and Juan Rubio-Ramírez. 2012. "The term structure of interest rates in a DSGE model with recursive preferences". *Journal of Monetary Economics* 59, 634-648.

- Vázquez, Jesús, and Aguilar, Pablo. 2021. "Adaptive learning with term structure information." *European Economic Review* 134, May, article 103689.

- Vázquez, Jesús, Ramón María-Dolores, and Juan M. Londoño. 2013. "On the informational role of term structure in the US monetary policy rule." *Journal of Economic Dynamics and Control* 37, 1852-1871.

- Walsh, Carl E. 2010. *Monetary Theory and Practice*. The MIT Press: Cambridge, Massachusetts.

- Wu, Jing. C., and Fan D. Xia. 2016. Measuring the macroeconomic impact of monetary policy at the zero lower bound." *Journal of Money, Credit, and Banking* 48, 253-291.

- Wu, Jing. C., and Ji Zhang. 2019. "A shadow rate new Keynesian model." *Journal of Economic Dynamics and Control* 107, October, article 103728.